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University of Ottawa
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School of Electrical Engineering
and Computer Science

ELG 3126 **RANDOM SIGNALS AND SYSTEMS** **Winter 2023**

ASSIGNMENT 2

(due at 8:30 AM Thursday, Jan. 26, submission via BrightSpace)

1. A random experiment consists of drawing two balls in succession and "at random" from an urn containing five black balls and one white ball and observing the result by colour.
   (a) Specify a sample space for this experiment.
   (b) Suppose the experiment was modified so that after the first ball is selected, it is returned to the urn before the second ball is drawn. What is the sample space now?
   (c) If the experiment was conducted repeatedly, what would the relative frequency of the result being two white balls be in the long run? Give the answer for the experiment as in (a) and also as in (b)?
   (d) Does the outcome of the second draw from the urn depend in some way on the outcome of the first draw in either of the two experiments?
2. Three balls numbered 1, 2 and 3 in a box are drawn at random one at a time until the box is empty. The sequence of the balls drawn is noted.
   (a) Find the sample space of the experiment.
   (b) Find the event $\mathcal{A}_k$ corresponding the the statement that "the kth ball is selected in the kth or earlier draw," for $k = 1, 2, 3$.
   (c) Find the event $\mathcal{A}_1 \cap \mathcal{A}_2 \cap \mathcal{A}_3$ and describe the event in words.
   (d) Find the event $\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3$ and describe the event in words.
   (e) Find the event $\overline{\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3}$ and describe the event in words.
3. A regular six-sided die is tossed and the result of the number of dots on the side facing up is noted as the outcome of the experiment.
   (a) Find the probability of the elementary events under the assumption that all outcomes are equally probable.
   (b) Find the probability of the elementary events under the assumption that all outcomes are equally probable except that the probability of $\{3\}$ is triple that of the others.
   (c) Find the probability of an odd outcome under the assumptions in (a) and (b).
4. A single card is drawn fairly from a 52 card deck of common playing cards.
   (a) Find the probability that a face card (Jack, Queen or King) is drawn.
   (b) Find the probability of a numbered card less than 5 is drawn.
   (c) Find the probability that the card is a 5 and has a black suit.
5. Prove $\mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2) \geq \max\{\mathcal{P}(\mathcal{A}_1), \mathcal{P}(\mathcal{A}_2)\}$, and that $\mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3) \geq \max\{\mathcal{P}(\mathcal{A}_1), \mathcal{P}(\mathcal{A}_2), \mathcal{P}(\mathcal{A}_3)\}$. How does this generalize to N events?
   Hint: Prove first that $\mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2) \geq \mathcal{P}(\mathcal{A}_1)$ and $\mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2) \geq \mathcal{P}(\mathcal{A}_2)$.
6. Prove that $\mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3) \leq \mathcal{P}(\mathcal{A}_1) + \mathcal{P}(\mathcal{A}_2) + \mathcal{P}(\mathcal{A}_3)$. This is the *union bound* (also known as the *subadditivity property* of probability) for three events. How does this generalize to N events?

. . . *(over)*

7. A certain pack of cards contains one red card numbered as 1, four white cards numbered 1 to 4, and five black cards numbered 1 to 5. How many ways are there of arranging these ten cards in a row? Supposed we draw the cards at random and place them in a row. What is the probability that every second card is black? What is the probability that every second card is not black?

8. You win the "6/49" lottery if you correctly predict which six of 49 numbers are drawn when six numbered balls are randomly chosen from an urn containing 49 balls numbered 1 through 49. The balls are chosen without replacement and the order of selection does not matter. Find the probability of winning the lottery. Does it depend on the choice you make of the numbers? What is the probability of correctly guessing at least four of the numbers correctly?

9. Show that conditional probability satisfies the basic axioms of probability, i.e., that for all events $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ where $\mathcal{P}(\mathcal{B}) > 0$,
   (i) $\mathcal{P}(\mathcal{A} \mid \mathcal{B}) \geq 0$,
   (ii) $\mathcal{P}(\mathcal{S} \mid \mathcal{B}) = 1$,
   (iii) if $\mathcal{A}$ and $\mathcal{C}$ are disjoint, $\mathcal{P}(\mathcal{A} \cup \mathcal{C} \mid \mathcal{B}) = \mathcal{P}(\mathcal{A} \mid \mathcal{B}) + \mathcal{P}(\mathcal{C} \mid \mathcal{B})$.

10. A manufacturer uses components from three sources. Components from source A, from source B and from source C are defective with probabilities 0.002, 0.005 and 0.001 respectively. A randomly chosen component in a manufactured item is equally likely to have come from each source. If a component in a manufactured item is found to be defective, what is the probability it came from source A? from source C?

11. Suppose we select two distinct numbers at random from the list $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and add these two numbers together. What is the probability that this sum is less than or equal to 2 plus the last digit of your student number?

12. A random experiment consists of tossing a coin, followed by more coin tosses depending on the results of the first toss. The coin is fair, but it is deemed to possibly come up "Heads," Tails," or "edge"" even though the last possibility has a 0 frequency of occurrence; the probabilities of the three possibilities for the first coin toss are $\frac{1}{2}$, $\frac{1}{2}$ and 0. When the first coin toss is "Heads," the coin is tossed again thrice; if the first coin toss is "Tails," the coin is tossed again twice; if the first coin toss is "edge," the coin is tossed again once. Each toss is made independently of any other. Find (i) the conditional probability that there are two "Heads" from coin tosses given that the first toss was "Heads," and (ii) the conditional probability that there is one "Heads" from coin tosses given that the first toss was "edge."
    Hint: The second question (ii) answer is not $\frac{1}{2}$.

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ASSIGNMENT 2

SOLUTIONS

1/ (a) The result of the experiment may be described by an ordered pair of colours denoting the colour of the first ball drawn and then the colour of the second ball. The possibilities are (black, black), (black, white) and (white, black), so $\mathcal{S} = \{(\text{black}, \text{black}), (\text{black}, \text{white}), (\text{white}, \text{black})\}$.

(b) In this scenario, it becomes possible to draw two white balls in succession, so now $\mathcal{S} = \{(\text{black}, \text{black}), (\text{black}, \text{white}), (\text{white}, \text{black}), (\text{white.white})\}$.

(c) In the (a) scenario, drawing two white balls is not a possible outcome and so its frequency of occurrence would be 0. In the (b) scenario, since the chance a white ball is drawn each time is clearly $\frac{1}{6}$, drawing a white ball in each of two independent draws (each draw is an independent experiment) is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

(d) In the (a) style experiment, if the first draw is white, the second draw must be black, while if the first ball is black, the second draw is four times more likely to be black as white. In the (b) style experiment, as the urn always contains the same six balls for each draw (five black and one white), the results of the first draw cannot influence the second draw in any sense.

2/ (a) Clearly $\mathcal{S} = \{(1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), (3,2,1)\}$.

(b) $\mathcal{A}_1 = \{(1,2,3), (1,3,2)\}$. $\mathcal{A}_2 = \{(1,2,3), (2,1,3), (2,3,1), (3,2,1)\}$. $\mathcal{A}_3 = \mathcal{S}$.

(c) $\mathcal{A}_1 \cap \mathcal{A}_2 \cap \mathcal{A}_3 = \{(1,2,3)\}$—balls are selected in their numbered order.

(d) $\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3 = \mathcal{S}$—balls may be selected in any order (the certain event).

(e) $\overline{\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3} = \varnothing$—the null event

3/ (a) There are six possible outcomes, so the probability of every singleton is $\frac{1}{6}$.

(b) If the probability of $\{3\}$ is x, then the probability of the other singletons is $\frac{1}{3}x$. Thus $x + 5 \times \frac{1}{3}x = \frac{8}{3}x = 1$, which gives us that $x = \frac{3}{8}$, i.e., $\mathcal{P}(\{3\}) = \frac{3}{8}$, and each of the other five singletons has probability $\frac{1}{8}$.

(c) $\mathcal{P}(\{1,3,5\}) = \mathcal{P}(\{1\}) + \mathcal{P}(\{3\}) + \mathcal{P}(\{5\})$, so in (a) this is $3 \times \frac{1}{6} = \frac{1}{2}$, while in (b) this is $\frac{1}{8} + \frac{3}{8} + \frac{1}{8} = \frac{5}{8}$.

4/ As there are 52 cards in the deck, then here each singleton has probability $\frac{1}{52}$.

(a) There are three types of cards to draw—J, Q, and K—in four suits, so again there are 12 cards that we can draw in this event, so the probability is $\frac{12}{52} = \frac{3}{13}$.

(b) There are three types of cards to draw—2, 3, and 4—in four suits, so again there are 12 cards that we can draw in this event, so the probability is $\frac{12}{52} = \frac{3}{13}$.

(c) There are two black 5 cards, so the probability is $\frac{2}{52} = \frac{1}{26}$.

. . . *(continued on the next page)*

5/ *Proof*: $\mathcal{A}_1 \cup \mathcal{A}_2 = \mathcal{A}_1 \cup (\mathcal{A}_2 - \mathcal{A}_1)$, where $\mathcal{A}_1$ and $(\mathcal{A}_2 - \mathcal{A}_1)$ are disjoint events. Hence $\mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2) = \mathcal{P}(\mathcal{A}_1) + \mathcal{P}(\mathcal{A}_2 - \mathcal{A}_1))$ (by Axiom III). But by Axiom I, $\mathcal{P}(\mathcal{A}_2 - \mathcal{A}_1) \geq 0$ so $\mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2) \geq \mathcal{P}(\mathcal{A}_1)$. Interchanging $\mathcal{A}_1$ and $\mathcal{A}_2$ in this gives us that also $\mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2) \geq \mathcal{P}(\mathcal{A}_2)$. Hence $\mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2)$ is at least as large as both $\mathcal{P}(\mathcal{A}_1)$ and $\mathcal{P}(\mathcal{A}_2)$ giving us that

$$\mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2) \geq \max\{\mathcal{P}(\mathcal{A}_2), \mathcal{P}(\mathcal{A}_2)\}.$$

Following the same reasoning,

$$\begin{aligned} \mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3) &= \mathcal{P}(\mathcal{A}_1 \cup [\mathcal{A}_2 \cup \mathcal{A}_3]) \geq \mathcal{P}(\mathcal{A}_1); \\ \mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3) &= \mathcal{P}(\mathcal{A}_2 \cup [\mathcal{A}_1 \cup \mathcal{A}_3]) \geq \mathcal{P}(\mathcal{A}_2); \qquad \Longrightarrow \mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3) \geq \max\{\mathcal{P}(\mathcal{A}_1), \mathcal{P}(\mathcal{A}_2), \mathcal{P}(\mathcal{A}_3)\}. \\ \mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3) &= \mathcal{P}(\mathcal{A}_3 \cup [\mathcal{A}_2 \cup \mathcal{A}_1]) \geq \mathcal{P}(\mathcal{A}_3); \end{aligned}$$

Clearly this has the obvious generalization that for any N events $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3, \ldots, \mathcal{A}_N$,

$$\mathcal{P}(\bigcup_{i=1}^{N} \mathcal{A}_i) \geq \max\{\mathcal{P}(\mathcal{A}_1), \mathcal{P}(\mathcal{A}_2), \mathcal{P}(\mathcal{A}_3), \ldots, \mathcal{P}(\mathcal{A}_N)\} \qquad \text{for all finite } N.$$

6/ *Proof*: We showed in class that for arbitrary events $\mathcal{B}$ and $\mathcal{C}$, $\mathcal{P}(\mathcal{B} \cup \mathcal{C}) \leq \mathcal{P}(\mathcal{B}) + \mathcal{P}(\mathcal{C})$ $(\star)$.
Now then from this

$$\mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3) = \mathcal{P}(\mathcal{A}_1 \cup [\mathcal{A}_2 \cup \mathcal{A}_3]) \leq \mathcal{P}(\mathcal{A}_1) + \mathcal{P}(\mathcal{A}_2 \cup \mathcal{A}_3).$$

But from $(\star)$, $\mathcal{P}(\mathcal{A}_2 \cup \mathcal{A}_3) \leq \mathcal{P}(\mathcal{A}_2) + \mathcal{P}(\mathcal{A}_3)$ so

$$\mathcal{P}(\mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3) \leq \mathcal{P}(\mathcal{A}_1) + \mathcal{P}(\mathcal{A}_2) + \mathcal{P}(\mathcal{A}_3)$$

as required.

The generalization of this is that for any events $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3, \ldots, \mathcal{A}_N$,

$$\mathcal{P}(\bigcup_{i=1}^{N} \mathcal{A}_i) \leq \sum_{i=1}^{N} \mathcal{P}(\mathcal{A}_i). \qquad \text{for all finite } N. \tag{$\ddagger$}$$

Proof: We have already eplicitly established this statement for $N = 1$, 2, and 3. Now assume that the statement is true for $N = k$, that is $\mathcal{P}(\bigcup_{i=1}^{k} \mathcal{A}_i) \leq \sum_{i=1}^{k} \mathcal{P}(\mathcal{A}_i)$.Then for $N = k+1$ we have (by using the case of $N = 2$)

$$\mathcal{P}(\bigcup_{i=1}^{k+1} \mathcal{A}_i) = \mathcal{P}([\bigcup_{i=1}^{k} \mathcal{A}_i] \cup \mathcal{A}_{k+1}) \leq [\sum_{i=1}^{k} \mathcal{P}(\mathcal{A}_i)] + \mathcal{P}(\mathcal{A}_{k+1}).$$

The right hand side of this is of course $\sum_{i=1}^{k+1} \mathcal{P}(\mathcal{A}_i)$, us $\mathcal{P}(\bigcup_{i=1}^{k+1} \mathcal{A}_i) \leq \sum_{i=1}^{k+1} \mathcal{P}(\mathcal{A}_i)$. Thus by the principle of mathematical induction, we have establish the valididty of $(\ddagger)$

Remark: This proof does NOT establish the result holds for infinite N. We can prove the statement is also true for $N = \infty$, that is

$$\mathcal{P}(\bigcup_{i=1}^{\infty} \mathcal{A}_i) \leq \sum_{i=1}^{\infty} \mathcal{P}(\mathcal{A}_i),$$

by using the Continuity Theorem of Probability as follows: Let $\mathcal{B}_n = \bigcup_{i=1}^{n} \mathcal{A}_i$ for $n = 1, 2, 3, \ldots$. This is a monotonic increasing sequence whose limit is $\bigcup_{i=1}^{\infty} \mathcal{A}_i$. Thus the Continuity Theorem states that

$$\mathcal{P}(\bigcup_{i=1}^{\infty} \mathcal{A}_i) = \lim_{n \to \infty} \mathcal{P}(\mathcal{B}_n).$$

. . . *(problem continued on the next page)*

6/ *(continued)*

But the above shows $\mathcal{P}(\mathcal{B}_n) \leq \sum_{i=1}^{n} \mathcal{P}(\mathcal{A}_i)$ for all n, so

$$\lim_{n\to\infty} \mathcal{P}(\mathcal{B}_n) \leq \lim_{n\to\infty} \sum_{i=1}^{n} \mathcal{P}(\mathcal{A}_i),$$

which is the statement that

$$\mathcal{P}(\bigcup_{i=1}^{\infty} \mathcal{A}_i) \leq \sum_{i=1}^{\infty} \mathcal{P}(\mathcal{A}_i).$$

7/ If the cards were distinct, then there would be $10! = 3628800$ ways of arranging the cards, but if we are noting only colours, then the number of different outcomes (arrangements of colours) is 10! reduced by a factor $k!$ for each group of k cards of a single colour. Here then there are $10!/(1!4!5!) = 1260$ ways of arranging the coloured cards.

Although there are three colours, for the purposes of the second part of this question we can say there are five black cards and five non-black cards. Placement of the cards can be described as a selection process where we select five positions from ten for the black cards to occupy, and the other cards filling the other positions. Viewing the outcomes this way, we see there are $\binom{10}{5} = \frac{10!}{(5!)^2} = 252$ outcomes in the sample space. Of these, two correspond to required alternating positions, so the probability that every second card in the list is black is $\frac{2}{252} = \frac{1}{126} \simeq 7.937 \times 10^{-3}$.

An alternate approach to solve this is to regard the outcome as a sequence of choices for placing 10 distinct cards, first placing the black cards, then the red ones and finally the white ones. Then there are 10! outcomes. If we want to start the arrangement with a black card and then have alternating non-black and black, there are $5! \times 5!$ ways to do so, and an equal number that start with a non-black card and then have alternating non-black and black. Hence now there are $2 \times (5!)^2$ outcomes in the event, so the probability is $\frac{2\times(5!)^2}{10!} = \frac{1}{126}$.

There is symmetry in the experiment between black and "non-black" cards (five of each), so the probability that ever second card is non-black must be the same as the probability that every second card is black...it is $\frac{1}{126}$.

8/ There are $\binom{49}{6} = \frac{49!}{6!43!} = 13,983,816$ different possible choices of six numbers. Hence the probability of winning the lottery is $\frac{1}{13983816} \simeq 7.15 \times 10^{-8}$, and this does not depend on what selection of numbers is made—every choice of six numbers has the same chance of winning,

To calculate the probability of matching at least four of the six chosen numbers we note that there are $\binom{6}{4} = 15$ subsets of four items from a set of six items. If we want four selected numbers to match and not a fifth or sixth selected number, the number of choices we have for the fifth and six numbers to be drawn is $\binom{49-6=43}{2} = 903$, so the number of draws of balls that produces a match of exactly four from any preselected set of of six chosen numbers is $15 \times 903 = 13{,}545$. (For example, if you pick to play $\{1, 2, 3, 4, 5, 6\}$, there are 903 sets that match 1,2,3,4 and not 5,6, there are 903 sets that match 1,2,3,5 and not 4,6, etc.) Likewise there are $\binom{6}{5} = 6$ subsets of five items from a set of six items. If we want five selected numbers to match and not a sixth selected number, the number of choices we have for six numbers to be drawn is 49-6=43, so the number of draws of balls that produces a match of exactly five from any preselected set of of six chosen numbers is $6 \times 43 = 258$. There is only one way to match all six chosen numbers. Thus the number of draws that produce a match of *at least* four is $1 + 258 + 13{,}545 = 13{,}804$. Hence the probability of picking six numbers and matching at least four of the lottery numbers is

$$\frac{13{,}804}{13{,}983{,}816} \simeq 9.8714 \times 10^{-4}.$$

. . . *(continued on the next page)*

9/ We have that by definition (for $\mathcal{P}(\mathcal{B}) > 0$) that $\mathcal{P}(\mathcal{A} \mid \mathcal{B}) = \dfrac{\mathcal{P}(\mathcal{A} \cap \mathcal{B})}{\mathcal{P}(\mathcal{B})}$.

(i) Since $\mathcal{A} \cap \mathcal{B}$ is an event, from the first axiom of probobility, $\mathcal{P}(\mathcal{A} \cap \mathcal{B}) \geq 0$. Since $\mathcal{P}(\mathcal{B}) > 0$, $\mathcal{P}(\mathcal{A} \mid \mathcal{B}) \geq 0$.

(ii) Since $\mathcal{S} \cap \mathcal{B} = \mathcal{B}$, $\mathcal{P}(\mathcal{S} \mid \mathcal{B}) = \dfrac{\mathcal{P}(\mathcal{S} \cap \mathcal{B})}{\mathcal{P}(\mathcal{B})} = \dfrac{\mathcal{P}(\mathcal{B})}{\mathcal{P}(\mathcal{B})} = 1$.

(iii) If $\mathcal{A}$ and $\mathcal{C}$ are disjoint, and $(\mathcal{A} \cup \mathcal{C}) \cap \mathcal{B} = (\mathcal{A} \cap \mathcal{B}) \cup (\mathcal{C} \cap \mathcal{B})$ where $\mathcal{A} \cap \mathcal{B}$ and $\mathcal{C} \cap \mathcal{B}$ are disjoint, we have that

$$\mathcal{P}(\mathcal{A} \cup \mathcal{C} \mid \mathcal{B}) = \frac{\mathcal{P}((\mathcal{A} \cup \mathcal{C}) \cap \mathcal{B})}{\mathcal{P}(\mathcal{B})} = \frac{\mathcal{P}(\mathcal{A} \cap \mathcal{B}) + \mathcal{P}(\mathcal{C} \cap \mathcal{B})}{\mathcal{P}(\mathcal{B})} = \frac{\mathcal{P}(\mathcal{A} \cap \mathcal{B})}{\mathcal{P}(\mathcal{B})} + \frac{\mathcal{P}(\mathcal{C} \cap \mathcal{B})}{\mathcal{P}(\mathcal{B})} = \mathcal{P}(\mathcal{A} \mid \mathcal{B}) + \mathcal{P}(\mathcal{C} \mid \mathcal{B})$$

as required.

10/ The probability that a randomly selected component would be defective is

$$\begin{aligned} &\mathcal{P}(\text{defective} \mid \text{from A})\mathcal{P}(\text{from A}) + \mathcal{P}(\text{defective} \mid \text{from B})\mathcal{P}(\text{from B}) + \mathcal{P}(\text{defective} \mid \text{from C})\mathcal{P}(\text{from C}) \\ &= \frac{0.002 + 0.005 + 0.001}{3} = \frac{0.008}{3} \simeq 0.00267. \end{aligned}$$

Then

$$\begin{aligned} \mathcal{P}(\text{from A} \mid \text{defective}) &= \frac{\mathcal{P}(\text{defective} \mid \text{from A})\mathcal{P}(\text{from A})}{\mathcal{P}(\text{defective})} = \frac{0.002/3}{0.008/3} = \tfrac{1}{4}; \\ \mathcal{P}(\text{from C} \mid \text{defective}) &= \frac{\mathcal{P}(\text{defective} \mid \text{from C})\mathcal{P}(\text{from C})}{\mathcal{P}(\text{defective})} = \frac{0.001/3}{0.008/3} = \tfrac{1}{8}. \end{aligned}$$

11/ There are $\binom{10}{2} = 10 \times 9/2 = 45$ different possible sets of two numbers from this list with their sum ranging from 1 to 17. Clearly each singleton (subset of two elements from $\{0, 1, 2, 3, \ldots, 9\}$) in this situation is equiprobable. Now for each possible last digit of a student number:

(i) If the last digit of your student number is 0 then all choices have a sum of 2 greater the last digit except $\{0, 1\}, \{0, 2\}$, so just two outcomes meet the criterion. Thus the probability in question is $\frac{2}{45}$.

(ii) If the last digit of your student number is 1, then four of the possible sets meets the criterion, so then the probability in question is $\frac{4}{45}$.

(iii) If the last digit of your student number is 2, then six of the possible sets meets the criterion, so then the probability in question is $\frac{6}{45} = \frac{2}{15}$.

(iv) If the last digit of your student number is 3, then nine of the possible sets meets the criterion, so then the probability in question is $\frac{9}{45} = \frac{1}{5}$.

(v) If the last digit of your student number is 4, then twelve of the possible sets meets the criterion, so then the probability in question is $\frac{12}{45} = \frac{4}{15}$.

(vi) If the last digit of your student number is 5, then sixteen of the possible sets meets the criterion, so then the probability in question is $\frac{16}{45}$.

(vii) If the last digit of your student number is 6, then twenty of the possible sets meets the criterion, so then the probability in question is $\frac{20}{45} = \frac{4}{9}$.

(viii) If the last digit of your student number is 7, then twenty-five of the possible sets meets the criterion, so then the probability in question is $\frac{25}{45} = \frac{5}{9}$.

(ix) If the last digit of your student number is 8, then twenty-nine of the possible sets meets the criterion, so then the probability in question is $\frac{29}{45}$.

(x) If the last digit of your student number is 9, then thirty-three of the possible sets meets the criterion, so then the probability in question is $\frac{33}{45} = \frac{11}{15}$.

. . . (continued on the next page)

12/ It is quite tempting to answer this question quickly, reasoning in (i) that the conditional probability is just the probability of the coin coming up "Heads" exactly once in the next three tosses after the first, which is the probability of HTT, THT or TTH—three out of eight equally probable possibilities and so obviously is $\frac{3}{8}$. For (ii) we might similarly reason that since we toss the coin once if the first is an edge, the conditional probability is the probability that a fair coin tossed once comes up with a "Heads" and so the conditional probability is $\frac{1}{2}$. This latter answer is INCORRECT. The correct answer for (ii) is that because the conditioning event ("the first toss is an edge") has zero probability, the conditional probability sought in (ii) is *undefined*.