Assignment on Buckling of Columns

Starting with the differential equation of equilibrium for column buckling $EIy^{iv} + Py'' = 0$, and its solution $y(x) = A\cos kx + B\sin kx + Cx + D$, in which $k = \sqrt{P/EI}$, it is required to formulate the buckling load P for a column fixed at both ends.

Derivatives;

$$y = A\cos kx + B\sin kx + Cx + D$$
$$y' = -Ak\sin kx + Bk\cos kx + C$$

Boundry Conditions

fixed at $x = 0$

BC1: $y = 0 \leftarrow$ deflection
BC2: $y' = 0 \leftarrow$ Slope

fixed at $x = l$

BC3 ; $y = 0 \leftarrow$ deflection
BC4 ; $y' = 0 \leftarrow$ Slope

BC1 and BC2;

$$y(x=0) = 0 \Rightarrow y = A\cos k(0) + B\sin k(0) + C(0) + D = 0$$

$$\therefore \quad A + D = 0 \qquad (1)$$

$$y'(x=0) = 0 \Rightarrow y' = -Ak\sin(0) + Bk\cos(0) + C = 0$$

$$\therefore \quad Bk + C = 0 \qquad (2)$$

BC3 and BC4;

$$y(x=l) = 0 = A\cos kl + B\sin kl + Cl + D \qquad (3)$$

$$y'(x=l) = 0 = -Ak\sin kl + Bk\cos kl + C \qquad (4)$$

Matrix Form

$$\begin{bmatrix} \cancel{1} & \cancel{0} & \cancel{0} & \cancel{1} \\ 0 & k & 1 & 0 \\ \cos kl & \sin kl & 0 & 0 \\ -k\sin kl & k\cos kl & 1 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Trivial $\quad A = B = C = D = 0$

find Det;

$$0 = 1 \begin{bmatrix} k & 1 & 0 \\ \cancel{\sin kl} & \cancel{0} & \cancel{0} \\ k\cos kl & 1 & 0 \end{bmatrix} - 1 \begin{bmatrix} \cancel{0} & \cancel{k} & \cancel{1} \\ \cos kl & \sin kl & 0 \\ -k\sin kl & k\cos kl & 1 \end{bmatrix}$$

$$0 = \sin kl \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} - \left(-k \begin{bmatrix} \cos kl & 0 \\ -k\sin kl & 1 \end{bmatrix} + 1 \begin{bmatrix} \cos kl & \sin kl \\ -k\sin kl & k\cos kl \end{bmatrix} \right)$$

$$0 = 0 + k(\cos kl) - k\left(\overbrace{\cos^2 kl + \sin^2 kl}^{1}\right)$$

$$0 = k\cos kl - k$$

$$0 = (\cos kl - 1)\, k$$

<u>$k = 0$ is trivial</u>

$\therefore$

$$\cos kl = 1 \quad \Rightarrow \quad kl = 2n\pi$$

$\therefore \quad k = \dfrac{2n\pi}{l} \qquad$ Sub into $\quad k = \sqrt{\dfrac{P}{EI}}$

$$\boxed{\therefore \quad P = \frac{4n^2\pi^2 EI}{l^2}}$$