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Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

Discrete Math for Computing MAT 1348 A

Midterm 1

February 6, 2023

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You must **sign below** to confirm that you have read, understand, and will follow these **instructions:**

- This is an 75-minute **closed-book** exam; no notes are allowed. **Calculators and notes are not permitted.**
- The exam consists of 8 questions on 10 pages, with a maximum of 25 points. Page 9 contains a reference table, and page 10 provides additional work space. If you need more additional space, you can use the backs of any of the pages. You may detach pages 9 and 10 for your convenience, but **do not detach any other pages.**
- Questions 1–2 comprise eight true or false questions worth 1 point each. Circle the correct answer. There is no penalty for an incorrect answer.
- Questions 3–5 are short-answer questions worth points as indicated. Write your answer in the box provided. Any rough work will not be graded.
- Questions 6–8 are long-answer questions worth points as indicated. You must use the technique that the question asks for and show all relevant steps in order to obtain full marks.
- **Cellular phones** and other electronic devices **are not permitted** during this exam. Phones and other devices must be turned off completely and stored out of reach. Do not keep them in your possession, such as in your pockets. If you are caught with such a device, the following may occur: academic fraud allegations will be filed which may result in your obtaining a 0 (zero) for the exam.

LAST NAME:____________________

First name:____________________

Student Number:____________________

Signature:____________________

Please circle the DGD section where you would like to have your test returned to you:

DGD 1	DGD 2	DGD 3	DGD 4
TUE 16:00 – 17:20	TUE 17:30 – 18:50	TUE 17:30 – 18:50	TUE 19:00 – 20:20
Kianoosh	Maxime	Xin Yi	Xin Yi

True/False Questions

*In Questions 1-2, **circle** T or F. Each correct answer is worth 1 point, and an incorrect answer is worth 0 points. You do not need to justify your answers.*

Q1. (a) In order to prove $P \to Q$ using a direct proof, you should assume P, then prove Q follows. T F

(b) $P \to Q$ is logically equivalent to its converse. T F

(c) If $\{P_1, P_2, P_3\}$ is an inconsistent set of propositions, then $P_1 \vee P_2 \vee P_3 \equiv F$. T F

(d) When you grow a truth tree with $\neg P$ at its root and all branches end up closed, then P is a contradiction. T F

Q2. Let X, Y, and Z be atoms and P_1, P_2, P_3, and C be compound propositions that depend on X, Y, and Z, as described in the following truth table:

X	Y	Z	P_1	P_2	P_3	C	$P_1 \oplus P_3$	$(P_1 \oplus P_3) \vee \neg P_2$
T	T	T	T	T	F	T	T	T
T	T	F	T	F	T	F	F	T
T	F	T	F	F	T	T	T	T
T	F	F	F	T	T	F	T	T
F	T	T	T	F	F	T	T	T
F	T	F	T	T	F	F	T	T
F	F	T	T	T	F	T	T	T
F	F	F	F	F	T	F	T	T

(a) $(P_1 \wedge P_2 \wedge P_3) \to C$ is a valid argument. T F

(b) The set $\{P_1, P_2, P_3\}$ is consistent. T F

(c) $(X \wedge Z) \vee (\neg X \wedge Z)$ is a DNF for C. T F

(d) $(P_1 \oplus P_3) \vee \neg P_2$ is a tautology. T F

Short-Answer Questions

Questions 3–5 are short-answer questions. Put your final answer in the box provided. You do not need to justify your answers.

Q3. (1 point) Complete the following definition:

The argument

$$\begin{array}{l} P_1 \\ P_2 \\ \underline{P_3} \\ \therefore C \end{array} \quad \textit{is } \textbf{valid } \textit{if...}$$

$(P_1 \land P_2 \land P_3) \to C$ is a tautology

Q4. (1 point) Suppose a, b, c are atoms and X is a compound proposition that is true if and only if a and b have opposite truth values. Write a DNF for X.

$(a \land \neg b) \lor (\neg a \land b)$

Q5. (1 point) Define the following propositions:

A : "Alex practises tennis everyday."

W : "Alex wins the tournament."

C : "The tournament is cancelled."

Use these variables with appropriate logical connectives to write a compound proposition that has the same meaning as the following sentence:

W A

"(In order for Alex to win the tournament, it is necessary that they practise tennis everyday) unless the tournament is cancelled."

C

$(W \to A) \lor C$

Long-Answer Questions

Questions 6–8 are long-answer questions. You must fully justify your work by showing all of your steps.

Q6. (4 points) Using the laws in the Table of Logical Equivalences (page 9), show that $X \equiv Y$, where X and Y are defined as follows:

$$X : \quad A \wedge (B \vee (C \to \neg A)) \qquad Y : \quad (\neg B \to \neg C) \wedge A$$

You must only use **one** law in each step, and you must not skip any steps. You must name the rule used at each step.

$A \wedge (B \vee (C \to \neg A))$

$\equiv A \wedge (B \vee (\neg C \vee \neg A))$ Implication law

$\equiv (A \wedge B) \vee (A \wedge (\neg C \vee \neg A))$ Distribution law

$\equiv (A \wedge B) \vee ((A \wedge \neg C) \vee (A \wedge \neg A))$ Distribution law

$\equiv (A \wedge B) \vee ((A \wedge \neg C) \vee F)$ negation law

$\equiv (A \wedge B) \vee (A \wedge \neg C)$ Identity law

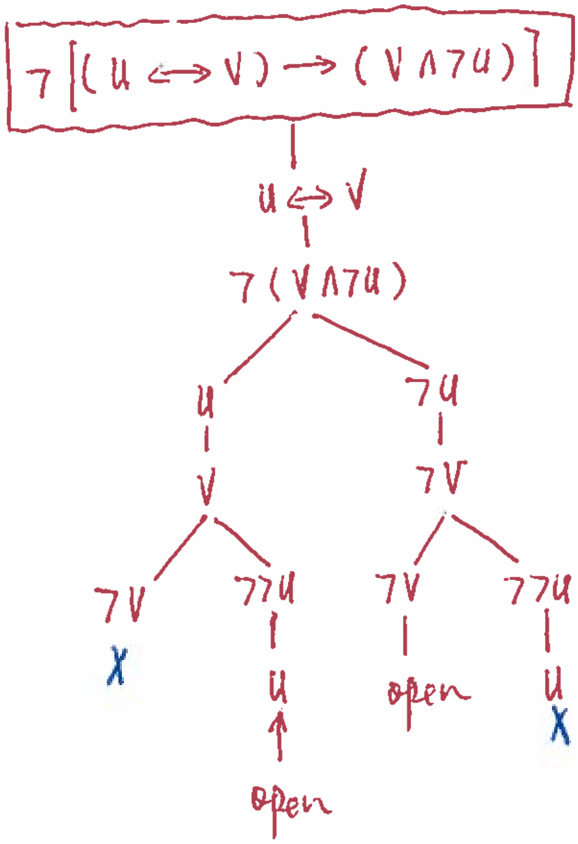
$\equiv A \wedge (B \vee \neg C)$ Distribution law

$\equiv (B \vee \neg C) \wedge A$ Commutativity law

$\equiv (\neg B \to \neg C) \wedge A$ Implication law

Q7. Define the compound proposition P : $(u \leftrightarrow v) \rightarrow (v \wedge \neg u)$.

(a) (3 points) Use a **truth tree** to determine whether P is a **tautology** and fill in your answers in the box below.



Is P a **tautology**? *Circle:* YES NO

If you answer YES, briefly explain why, making reference to your tree.

If you answer NO, give **one** counterexample and explain how you got it from your tree.

Counterexample : $u = T$, $v = T$ form a complete active path.

(b) (3 points) Using the same proposition P as in the previous question (P : $(u \leftrightarrow v) \rightarrow (v \wedge \neg u)$), use a **truth table** to determine whether P is a contradiction, a contingency, or a tautology, and fill in your answers in the box below.

u	v	$u \leftrightarrow v$	$v \wedge \neg u$	$(u \leftrightarrow v) \rightarrow (v \wedge \neg u)$
T	T	T	F	F
T	F	F	F	T
F	T	F	T	T
F	F	T	F	F

What kind of proposition is P?

Circle: **Contradiction** **Contingency** **Tautology**

Explain your answer by referencing specific rows or ~~columns in~~ your truth table.

The column for P has at least one T and at least one F.

Q8. (4 points) While visiting the Island of Knights & Knaves, you encounter inhabitants A, B, and C.

A says: "Greetings! The three of us are all knights."

B says: "No, A and I are of the same type, but C isn't the same type as us."

(Recall that "being the same type" means they are both knights or both knaves.)

Is C a knight, a knave, or are we unable to tell? Use any method we have learned in class and fully explain your reasoning.

Define a: "A is a knight" A says: $a \wedge b \wedge c$

b: "B is a knight" B says: $(a \leftrightarrow b) \wedge \neg(a \leftrightarrow c)$

c: "C is a knight"

a	b	c	A says $a \wedge b \wedge c$	B says $(a \leftrightarrow b) \wedge \neg(a \leftrightarrow c)$
T	T	T	T	F
T	T	F	F	T
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

only row where both match ∴ C is a knave

If A is a knight, then A's statement that all three are knight is True. But B's statement contradicts A's statement, a contradiction. Hence A is a knave.

If B is a knight, then B's statement says that B is a knave (the same as A), a contradiction, so B is a knave.

Then C must be a knave, otherwise B's statement would be true.