Quiz 2 solution

1. An ideal gas initially at 500 K and 8 bar undergoes a four-step mechanically reversible cycle in a closed system. In step 1-2, pressure decreases isothermally to 3 bar; in step 2-3, pressure decrease at constant volume to 2 bar; in step 3-4, volume decreases at constant pressure; and in step 4-1, the gas returns adiabatically to its initial state. Take $C_p= (7/2)$ R and $C_v = (5/2)$ R.

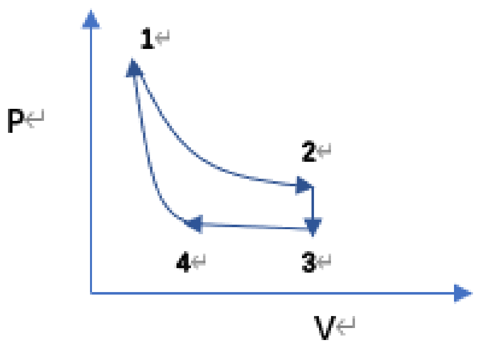
(a) Sketch the cycle on a PV diagram.

(b) Determine T, P, V for states 1, 2, 3, and 4 where unknown.

(c) Calculate q, w, ΔU and ΔH for each step of the cycle.

Answer:

(a) Sketch the cycle on a PV diagram.



(b) Determine (where unknown) both T and P for states 1, 2, 3, and 4.

C_p=7/2R and C_v=5/2R

At Point 1, P_1=8 bar T_1=500 K,

$$V_1 = \frac{RT_1}{P_1} = \frac{8.314\ Pa.m^3mol^{-1}K^{-1}*500K}{8\ bar*10^5\ Pa/bar} = 5.196\times10^{-3}\ \text{m}^3/\text{mol}$$

For isothermal expansion from Point 1 to Point 2, P_2 = 3 bar and $T_2 = T_1$=500 K

$$V_2 = \frac{RT_2}{P_2} = \frac{8.314\ Pa.m^3mol^{-1}K^{-1}*500\ K}{3\ bar*10^5\ Pa/bar} = 0.014\ \text{m}^3/\text{mol}$$

For const-V cooling from Point 2 to Point 3, P_3=2 bar and $V_3=V_2$=0.014 m^3/mol

$$T_3 = \frac{P_3V_3}{R} = \frac{2*10^5Pa*0.014\ \text{m}^3\ \text{mol}^{-1}}{8.314\ Pa.\text{m}^3\text{mol}^{-1}\text{K}^{-1}} = 333.33\ \text{K}$$

For const-P cooling from Point 3 to Point 4, $P_4= P_3$=2 bar

For adiabatic compression from Point 4 to Point 1,

$$T_4 = T_1(\frac{P_4}{P_1})^{\frac{R}{C_P}} = 500 * \left(\frac{2}{8}\right)^{\left(\frac{R}{3.5R}\right)} = 500 * (0.25)^{\left(\frac{1}{3.5}\right)} = 336.475 \text{ K}$$

$V_4 = RT_4/P_4 = 8.314*336.475/2*10^5 = 0.014$ m^3/mol

(c) Calculate Q, W, ΔU and ΔH for each step of the cycle.

Step12: Isothermal

ΔU_{12}=0 J/mol

ΔH_{12}=0 J/mol

$$q_{12} = -RT_1 \ln\left(\frac{P_2}{P_1}\right) = -8.314 * 500 * \ln\left(\frac{3}{8}\right) = 4077 \text{ J/mol}$$

w_{12}= -q_{12} = -4077 J/mol

Step23: Isochoric

$$\Delta U_{23} = C_v(T_3 - T_2) = \frac{5}{2} * 8.314 * (333.33 - 500) = -3.464\times10^3 \text{ J/mol}$$

$$\Delta H_{23} = C_p(T_3 - T_2) = \frac{7}{2} * 8.314 * (333.33 - 500) = -4.850\times10^3 \text{ J/mol}$$

w_{23}=0 J/mol

$q_{23}=\Delta U_{23}$= -3.464×10^3 J/mol

Step34: Isobaric

$$\Delta U_{34} = C_v(T_4 - T_3) = \frac{5}{2} * 8.314 * (336.475 - 333.33) = 65.37 \text{ J/mol}$$

$$\Delta H_{34} = C_p(T_4 - T_3) = \frac{7}{2} * 8.314 * (336.475 - 333.33) \quad \Delta H_{34} = 91.52 \text{ J/mol}$$

$$\Delta q_{34} = \Delta H_{34} = 91.52 \text{ J/mol}$$

$$w_{34} = -R(T_4 - T_3) = -8.314 * (336.475 - 333.33) = -26.148 \text{ J/mol}$$

Step41: Adiabatic

$$\Delta U_{41} = C_v(T_1 - T_4) = \frac{5}{2} * 8.314 * (500 - 336.475) = 3.399\times10^3 \text{ J/mol}$$

$$\Delta H_{41} = C_p(T_1 - T_4) = \frac{7}{2} * 8.314 * (500 - 336.475) = 4.758\times10^3 \text{ J/mol}$$

q_{41}=0 J/mol

$w_{41}=\Delta U_{41}= 3.399\times10^3$ J/mol