

MAT1322 Practice Midterm

INSTRUCTIONS FOR THIS MOCK TEST:

- ◇ Prepare your 1-page Cheat Sheet.
 - ◇ Get 8 pieces of blank paper (lined or graph paper is fine).
 - ◇ You may use a **basic scientific calculator**.
 - ◇ When ready, give yourself **70 minutes** to write the test. Other than your 1-page Cheat Sheet, this is a closed-book test.
 - ◇ **Goal for scanning and uploading:** After the 70 minutes is up, give yourself **10 minutes (max!)** to scan your work, including your Cheat Sheet, and create a single pdf copy. Practice the process you plan to use for the real midterm so that you can easily do this within a few minutes. Once you have created a pdf copy of your work, upload and submit your work in the Practice Brightspace Assignment.
- Detailed instructions for the format and technical requirements of Test 1 have been announced in Brightspace and the course syllabus. Read them and let your prof know **before the midterm** if you have any concerns.

● MULTIPLE-CHOICE QUESTIONS

Your answers to multiple-choice questions do not need to be justified. You may write your scrap work on your paper but it will not be graded. When you reach your answer, clearly indicate the question number and write the letter of your response beside the question number:

For example: (*write out your scrap work, but it will not be graded*)

(*clearly indicate your final choice*) Q1. [letter of your choice]

● LONG-ANSWER QUESTIONS

For long-answer questions, all of your work must be justified and your steps must be written in a clear and logical order. Clearly indicate Question numbers.

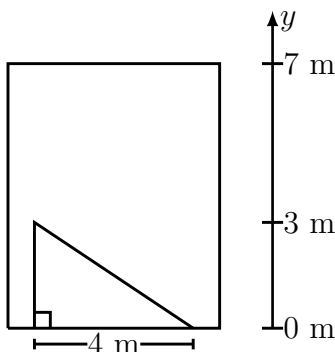
For example: Q5(b). [write a fully justified solution].

- **Scan-and-Upload Phase:** When the test-writing phase ends, you will be given 10 minutes to scan and create a pdf copy of your work which you will upload and submit in the Brightspace Assignment for Test 1.

MULTIPLE-CHOICE QUESTIONS

- Q1.** What is the area of the region bounded by the curves $y = 3x^2$ and $y + 2x = 1$ for $-1 \leq x \leq \frac{1}{3}$?
- A. $\frac{9}{8}$ B. $\frac{3}{8}$ C. $\frac{7}{9}$ D. $\frac{9}{10}$ E. $\frac{32}{27}$ F. $\frac{1}{4}$
- G. None of the above.

- Q2.** A reservoir has a triangular door located at the bottom of one of its vertical sides, as shown in the diagram below. The reservoir is 7 m high and filled to the top with water. The door is 4 m wide by 3 m high.



Let y represent the height from the bottom of the reservoir. Which of the following integrals represents the hydrostatic force exerted by the water on the door? Note that the density of water is 1000 kg/m^3 and the acceleration due to gravity is 9.8 m/s^2 .

- A. $9800 \int_0^3 \frac{4}{3}(3 - y)(y - 7)dy$ B. $9800 \int_0^3 6dy$ C. $9800 \int_0^3 2(7 - y)dy$
- D. $9800 \int_0^7 \frac{3}{4}(y - 7)dy$ E. $9800 \int_0^3 2(y - 7)dy$ F. $9800 \int_0^7 \frac{4}{3}(3 - y)dy$
- G. None of the above.

- Q3.** What is the arc length of the curve $y = 2x^{\frac{3}{2}} - 1$ between $x = 0$ and $x = 4$, rounded to one decimal place?
- A. 16.6 B. 25.2 C. 51.7 D. 74.4 E. 32.3 F. 54.9
- G. None of the above.

- Q4.** Use Euler's method with step size $h = 0.1$ to estimate $y(2.2)$, where $y(x)$ is the solution to the differential equation $y' = 2x^2 - y$ with initial condition $y(2) = 0$.
- A. 1.52 B. 0.93 C. 1.602 D. 1.428 E. 0.8 F. 1.742
- G. None of the above.

Q5. Let \mathcal{R} be the region bounded by the curve $y = e^{3x+1}$, the x -axis, and the lines $x = 2$ and $x = 5$.

Which of the following integrals represents the volume of the solid obtained by rotating the region \mathcal{R} about the x -axis?

- A. $\int_2^5 2\pi e^{6x+2} dx$ B. $\int_2^5 \pi e^{6x+2} dx$ C. $\int_2^4 \pi e^{6x} dx$
 D. $\int_2^5 \pi e^{(3x+1)^2} dx$ E. $\int_0^e \pi e^{3x+1} dx$ F. $\int_2^5 \pi e^{3x+1} dy$
 G. None of the above.

LONG-ANSWER QUESTIONS: Give detailed solutions, clearly showing each of your steps.

Q6. Consider the following differential equation: $\frac{dy}{dx} - y^2 + 4xy^2 = 0$

- i. Determine its general solution.
- ii. Determine the particular solution for which $y(0) = -2$.

Q7a. Consider the integral $\int_{-\infty}^5 \frac{x}{(x^2 + 1)^3} dx$.

Explain what makes this an improper integral. Determine whether it converges or diverges. Fully justify your answer using appropriate methods and notation. If it converges, find its exact value.

Q7b. Using the comparison test, determine whether the following improper integral converges or not:

$$\int_1^{\infty} \frac{\sin^2(x) + x}{x^3 + e^{5x}} dx$$

Q8. A pool as shown in the picture is filled with water. What work is done by pumping the water 1 m above the top of the pool? Clearly define all variables that enter into your solution and provide a diagram which shows their meaning.

Note that the density of water is 1000 kg/m^3 and the acceleration due to gravity is 9.8 m/s^2 .

