

Université d'Ottawa  
Faculté de génie

École de science informatique  
et de génie électrique



University of Ottawa  
Faculty of Engineering

School of Electrical Engineering  
and Computer Science

**ELG3106 Applied Electromagnetism Professor: H. Schriemer Date: Nov. 2<sup>nd</sup>, 2010**  
**ELG3506 Électromagnétisme Appliqué Professeure : K. Hinzer Date: 2 nov. 2010**

*Instructions*

- Allotted time : 80 minutes
- Closed book exam. Useful formulas are listed at the end of the exam.
- Programmable calculators are not authorized.
- Answer all questions. Explain clearly how you got to the final answer. Solutions without adequate will not be considered. Do not forget to indicate units.
- The mark for each question is indicated.
- Give back ALL material to the professor.

*Instructions*

- Temps alloué : 80 minutes
- Examen à livres fermés. Des formules utiles sont fournies à la fin de l'examen.
- Calculatrices pré-programmables non autorisées.
- Répondre à toutes les questions. Expliquer clairement comment vous êtes arrivés à la solution finale. Les solutions sans justification adéquate ne seront pas prises en compte. Ne pas oublier de donner les unités.
- Les valeurs des questions sont telles qu'indiquées.
- Remettre TOUT le matériel de la professeure.

Q1/	/ 8
Q2/	/ 10
Q3/	/ 2

Final Mark / Note finale:

**/20**

## Question 1

A uniform plane wave whose electric field is of the form

Une onde plane uniforme dont le champ électrique est donné par

$$\mathbf{E}(z, t) = \hat{\mathbf{a}}_x 7.5 e^{-0.004z} \cos(0.3\pi \times 10^8 t - 0.5\pi z) \quad \text{V/m}$$

is propagating in a homogeneous isotropic nonmagnetic medium.

se propage dans un milieu homogène, isotrope et non magnétique.

(a) Write the electric field in phasor notation.

Écrivez le champ électrique sous forme de phaseur.

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{a}}_x 7.5 e^{-0.004z} e^{-j0.5\pi z} = \hat{\mathbf{a}}_x E_0 e^{-\gamma z}$$

Therefore

$$E_0 = 7.5 \text{ V/m}$$

$$\gamma = \alpha + j\beta \Rightarrow \alpha = 0.004 \text{ Np/m} \quad \beta = 0.5\pi \text{ rad/m}$$

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(b) What are the wave frequency, the wavelength, and the phase velocity?

Quelles sont la fréquence, la longueur d'onde et la vitesse de phase de l'onde ?

$$\omega = 2\pi f = 0.3\pi \times 10^8 \text{ rad/s} \Rightarrow f = 0.15 \times 10^8 = 15 \text{ MHz}$$

$$\beta = \frac{2\pi}{\lambda} = 0.5\pi \text{ rad/m} \Rightarrow \lambda = \frac{2\pi}{0.5\pi} = 4 \text{ m}$$

$$u_p = \frac{\omega}{\beta} = \frac{0.3\pi \times 10^8}{0.5\pi} = 0.6 \times 10^8 \text{ m/s}$$

2

(c) What is the intrinsic impedance of the medium?

Quelle est l'impédance intrinsèque du milieu ?

$$\beta = \omega \sqrt{\mu \epsilon} \Rightarrow \epsilon = \frac{\beta^2}{\mu \omega^2} = \frac{(0.5\pi)^2}{(4\pi \times 10^{-7})(0.3\pi \times 10^8)^2} = 2.21 \times 10^{-10} \text{ F/m}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{4\pi \times 10^{-7}}{2.21 \times 10^{-10}}} = 75.4 \Omega$$

2

(d) What is the conductivity of the medium? Is the medium a low-loss medium, a good conductor, or somewhere in between? Justify your answer.

Quelle est la conductivité du milieu ? Est-ce un milieu à faibles pertes, un bon conducteur ou entre les deux ? Justifiez votre réponse.

It is not a good conductor because  $\alpha \neq \beta$

It is not lossless because  $\alpha \neq 0$

Assume it is low-loss and check:

$$\alpha = 0.5\sigma \sqrt{\mu/\epsilon} \Rightarrow \sigma = 2(0.004) \sqrt{\frac{2.21 \times 10^{-10}}{4\pi \times 10^{-7}}} = 1 \times 10^{-4}$$

$$\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\epsilon \omega} = \frac{1.06 \times 10^{-4}}{(2.21 \times 10^{-10})(0.3\pi \times 10^8)} = 5.1 \times 10^{-3} \ll 1$$

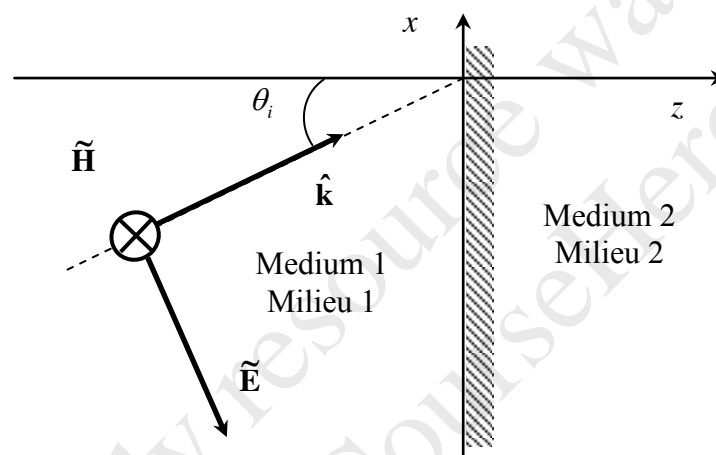
Yes, it is a low-loss medium.

2

## Question 2

Consider a uniform plane wave obliquely incident from medium 1 onto medium 2, as shown below; the incident angle is  $\theta_i = 30^\circ$ . The constitutive parameters of medium 1 are  $\epsilon_r = 4$  and  $\mu_r = 16$ , while those of medium 2 are  $\epsilon_r = 9$  and  $\mu_r = 27$ . The magnitude of the incident magnetic field is  $H_{io} = 0.1$  A/m, and its direction is  $-\hat{\mathbf{a}}_y$  as shown in the figure.

Soit une onde plane uniforme obliquement incidente d'un milieu 1 à un milieu 2 (voir figure ci-dessous). L'angle d'incidence est  $\theta_i = 30^\circ$ . Les paramètres du milieu 1 sont  $\epsilon_r = 4$  et  $\mu_r = 16$ , tandis que ceux du milieu 2 sont  $\epsilon_r = 9$  et  $\mu_r = 27$ . L'amplitude du champ magnétique incident est  $H_{io} = 0.1$  A/m et sa direction est  $-\hat{\mathbf{a}}_y$  comme montré sur la figure.



(a) Is this wave transverse electric or transverse magnetic? Justify your answer.

Cette onde est-elle transverse électrique ou magnétique ? Justifiez votre réponse.

This is a TM wave, since  $\mathbf{H}$  is normal to the plane of incidence

(b) Find the transmitted angle.

Déterminez l'angle transmis.

$$\begin{aligned}\sqrt{\mu_1 \epsilon_1} \sin \theta_i &= \sqrt{\mu_2 \epsilon_2} \sin \theta_t \\ \Rightarrow \sin \theta_t &= \sin(30^\circ) \sqrt{\frac{\mu_{r1} \epsilon_{r1}}{\mu_{r2} \epsilon_{r2}}} = (0.5) \sqrt{\frac{(16)(4)}{(27)(9)}} = (0.5)(0.5132) = 0.2566 \\ \text{Therefore } \theta_t &= 14.9^\circ\end{aligned}$$

2

(c) Find the reflection and transmission coefficients.

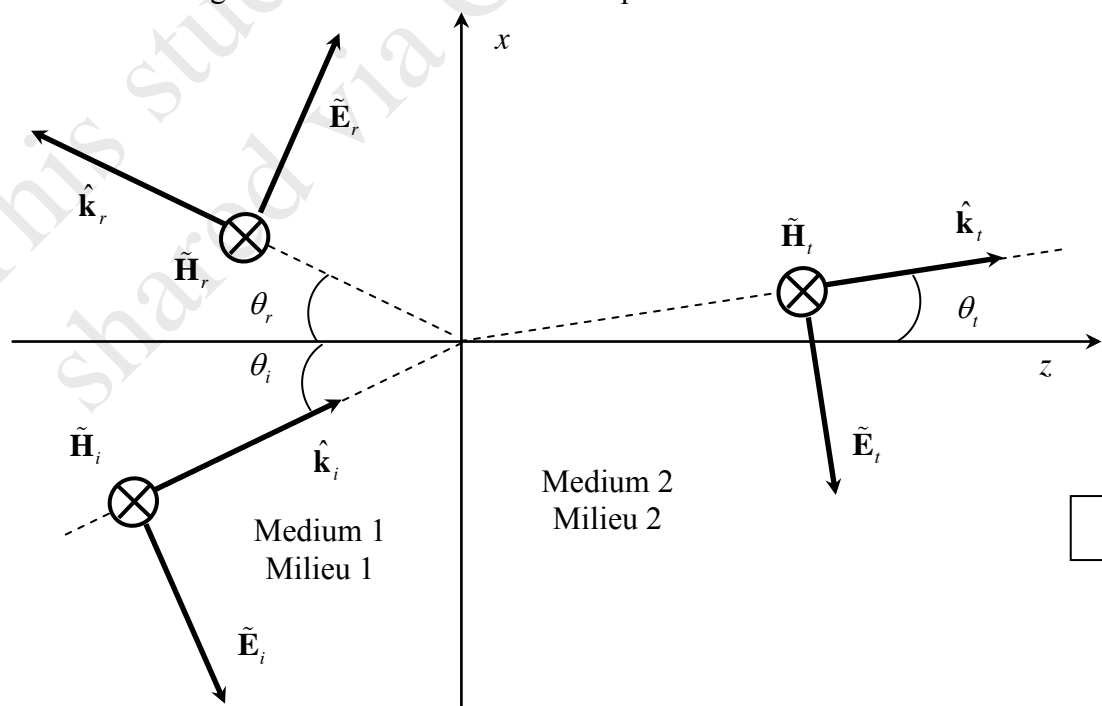
Déterminez les coefficients de réflexion et de transmission.

$$\begin{aligned}\Gamma_{\parallel} &= \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} = \frac{\sqrt{27/9} \cos(14.9^\circ) - \sqrt{16/4} \cos(30^\circ)}{\sqrt{27/9} \cos(14.9^\circ) + \sqrt{16/4} \cos(30^\circ)} \\ &= \frac{(1.7321)(0.9665) - (2)(0.866)}{(1.7321)(0.9665) + (2)(0.866)} = \frac{1.6741 - 1.732}{1.6741 + 1.732} = \frac{-0.0579}{3.4061} = -0.017 \\ \tau_{\parallel} &= \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} = \frac{2\sqrt{27/9} \cos(30^\circ)}{\sqrt{27/9} \cos \theta_t + \sqrt{16/4} \cos \theta_i} = \frac{3}{3.4061} = 0.88\end{aligned}$$

3

(d) Sketch the reflected and transmitted field directions on a diagram.

Dessinez sur un diagramme les directions des champs réfléchis et transmis.



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### Question 3

A plane wave in air with

Une onde plane dans l'air ayant

$$\mathbf{E}^i = \hat{\mathbf{y}} 20 e^{-j(3x+4z)} \quad \text{V/m}$$

$$\Gamma_{\perp} = -0.41$$

$$\theta_i = 36.87^\circ$$

is incident upon the planar surface of a dielectric material, with  $\epsilon_r = 4$ , occupying the half space  $z \geq 0$ .

est incidente sur une surface plane d'un diélectrique, ayant  $\epsilon_r = 4$ , occupant le demi- espace  $z \geq 0$ .

- (a) Find the reflected electric field in phasor form.

Trouvez l'expression du champ électrique réfléchi en forme phaseur.

Since/puisque  $\mathbf{E}_{\perp}^r = \hat{\mathbf{y}} \mathbf{E}_{\perp 0}^r e^{-jk_1(x \sin \theta_r - z \cos \theta_r)} \quad \text{V/m}$

and using the relation/et employant la relation  $E_0^r = \Gamma_{\perp} E_0^i$

$$\mathbf{E}^r = -\hat{\mathbf{y}} 8.2 e^{-j(3x-4z)}$$

1

Here we use the fact that  $\theta_i = \theta_r$  and that the  $z$ -direction has been reversed.

- (b) Find the reflected magnetic field in phasor form

Trouver l'expression du champ magnétique réfléchi en forme phaseur.

$$\mathbf{H}^r = -(\hat{\mathbf{x}} \cos \theta_i + \hat{\mathbf{z}} \sin \theta_i) \frac{8.2}{\eta_0} e^{-j(3x-4z)}$$

$$\mathbf{H}^r = -(\hat{\mathbf{x}} 17.4 + \hat{\mathbf{z}} 13.06) e^{-j(3x-4z)} \quad \text{mA/m}$$

1

# Equation sheet / Page de formules

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma| e^{j\theta_r} \quad S = \frac{|V_{\max}|}{|V_{\min}|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \nabla \cdot \mathbf{D} = \rho_v \quad \nabla \cdot \mathbf{B} = 0$$

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{k_1}{k_2} \quad \eta = \sqrt{\frac{\mu}{\varepsilon}} \quad k = \omega \sqrt{\mu \varepsilon}$$

$$\Gamma_{\perp} = \Gamma_{TE} = \frac{E_o^r}{E_o^i} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} \quad \tau_{\perp} = \tau_{TE} = \frac{E_o^{tr}}{E_o^i} = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t}$$

$$\Gamma_{\parallel} = \Gamma_{TM} = \frac{E_o^r}{E_o^i} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i} \quad \tau_{\parallel} = \tau_{TM} = \frac{E_o^{tr}}{E_o^i} = \frac{2\eta_2 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \quad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \quad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

Dans l'air/ in air  $\eta = 120 \pi = 377 \Omega$

$$\varepsilon_0 = 8.85 \times 10^{-12} \simeq \frac{1}{36\pi} \times 10^{-9} \text{ F/m} \quad \varepsilon = \varepsilon_r \varepsilon_0$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\varepsilon_c = \varepsilon - \frac{j\sigma}{\omega}$$

$$\nabla^2 \mathbf{E} + k_c^2 \mathbf{E} = \mathbf{0} \quad \nabla^2 \mathbf{H} + k_c^2 \mathbf{H} = \mathbf{0} \quad k_c = \omega \sqrt{\mu \varepsilon_c} \quad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

$$\tilde{\mathbf{S}}_{av} = \frac{1}{2} \text{Re}[\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*] \text{ (W/m}^2\text{)} \quad \tilde{\mathbf{S}}_{av}^i(z) = \hat{\mathbf{z}} \frac{|E_{i0}|^2}{2\eta_1^*}$$

$$\tilde{\mathbf{S}}_{1av}(z) = \hat{\mathbf{z}} \frac{|E_{i0}|^2}{2\eta_1^*} (1 - |\Gamma|^2) \quad \tilde{\mathbf{S}}_{2av}(z) = \hat{\mathbf{z}} \frac{|\tau|^2 |E_{i0}|^2}{2\eta_2^*}$$

$$\vec{H} = \frac{1}{\eta} \vec{a}_k \times \vec{E} \quad \lambda = \frac{2\pi}{k} \quad \omega = 2\pi f \quad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$S = \frac{|\tilde{\mathbf{E}}_i|_{\max}}{|\tilde{\mathbf{E}}_i|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad \tau = 1 + \Gamma$$

$$\underline{l_{\max} = \frac{\theta_r \lambda_1}{4\pi} + n \frac{\lambda_1}{2} \quad (\mathbf{n=0,1,2,...}) \quad \text{ou} \underline{\theta_r = 0} \quad \text{si} \underline{\eta_1 > \eta_2} \quad \text{et} \quad \underline{\theta_r = \pi} \quad \text{si} \underline{\eta_1 < \eta_2}$$

	<i>General Case</i> <i>Cas général</i>	<i>Lossless</i> <i>Milieu sans pertes</i>	<i>Low loss</i> <i>Milieu à faibles pertes</i>	<i>Good conductor</i> <i>Bon conducteur</i>
$\alpha$ (Np/m)	$\omega \left\{ \frac{1}{2} \mu \varepsilon' \left[ \sqrt{1 + \left( \frac{\varepsilon''}{\varepsilon'} \right)^2} - 1 \right] \right\}^{\frac{1}{2}}$	0	$\frac{\sigma}{2} \sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\pi f \mu \sigma}$
$\beta$ (rad/m)	$\omega \left\{ \frac{1}{2} \mu \varepsilon' \left[ \sqrt{1 + \left( \frac{\varepsilon''}{\varepsilon'} \right)^2} + 1 \right] \right\}^{\frac{1}{2}}$	$\omega \sqrt{\mu \varepsilon}$	$\omega \sqrt{\mu \varepsilon}$	$\sqrt{\pi f \mu \sigma}$
$\eta_c$ ( $\Omega$ )	$\sqrt{\frac{\mu}{\varepsilon'}} \left( 1 - j \frac{\varepsilon''}{\varepsilon'} \right)^{-\frac{1}{2}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$(1 + j) \frac{\alpha}{\sigma}$
$u_p$ (m/s)	$\frac{\omega}{\beta}$	$\frac{1}{\sqrt{\mu \varepsilon}}$	$\frac{1}{\sqrt{\mu \varepsilon}}$	$\sqrt{\frac{4\pi f}{\mu \sigma}}$