09/26/22
{1.2, 2.8, 3.6, 6.2, 6.6}

MAT 2379 - Assignment 1

1.2)

G: E = separated brows F = freckles

e = joined brows f = no freckles

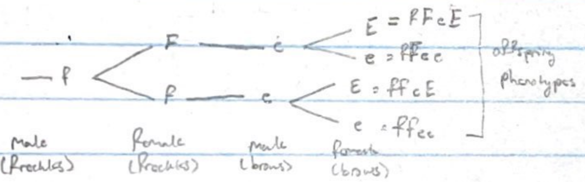
Woman: Ee Ff Man: ee ff

∴ Punnett Square:

male gamete	Female gamete $\frac{1}{4}$ EF	$\frac{1}{4}$ eF	$\frac{1}{4}$ Ef	$\frac{1}{4}$ ef
ef	$\frac{1}{4}$ EeFf	$\frac{1}{4}$ eeFf	$\frac{1}{4}$ Eeff	$\frac{1}{4}$ eeff
	separated brows, freckles	joined brows, freckles	separated brows, no freckles	joined brows, no freckles

∴ Probability of separated brows and freckles on offspring = $\frac{1}{4}$

Tree Diagram:



$\underline{\underline{\therefore}}$ yes, probability of separated brows + freckles in offspring = $\frac{1}{4}$.

2.8) G: total = 1150

C = 50 (A^+ in chemistry)

B = 375 (A^+ in Biology)

$C \cap B$ = 45 (A^+ in both C and B)

(a) $P(B \cap C')$ = A^+ in Biology (B), not in chemistry (C')

$P(B \cap C) = P(C \cap B)$

$P(B \cap C) = \frac{45}{1150}$

Let me try..

$P(B) = P(B \cap C) + P(B \cap C')$

$P(B) = \frac{375}{1150}$ $P(B \cap C) = \frac{45}{1150}$

$\therefore \frac{375}{1150} = \frac{45}{1150} + P(B \cap C')$

$\frac{375}{1150} - \frac{45}{1150} = P(B \cap C')$

$\frac{330}{1150} = P(B \cap C') = 0.287$

$= 0.29$

$\underline{\underline{\therefore}}$ $P(B \cap C') = 0.29$, or 0.287.

(b) $P(B' \cap C)$ = A^+ in chemistry (C), not in Biology (B')

$P(C \cap B') = P(B' \cap C)$

∴ this is like last part..

$P(C) = P(B \cap C) + P(C \cap B')$

$P(C) = \frac{50}{1150}$ $P(B \cap C) = \frac{45}{1150}$

$\therefore \frac{50}{1150} = \frac{45}{1150} + P(C \cap B')$

$\frac{50 - 45}{1150} = P(C \cap B')$

$\frac{5}{1150} = P(C \cap B')$

$0.00434 = P(C \cap B')$

$\underline{\underline{\therefore}}$ $P(C \cap B') = 0.00434$.

(e) $P(B' \cap C')$ = no A^+ in chemistry (C') or Biology (B')

lets recall..

$P(B' \cap C') = 1 - P(B \cup C)$

$P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$P(B \cup C) = \frac{375}{1150} + \frac{50}{1150} - \frac{45}{1150}$

$P(B \cup C) = \frac{425}{1150} - \frac{45}{1150}$

$P(B \cup C) = \frac{380}{1150}$

$\therefore P(B' \cap C') = 1 - P(B \cup C)$

$= 1 - \frac{380}{1150}$

$= \frac{1150 - 380}{1150}$

$P(B' \cap C') = \frac{770}{1150} = 0.669$

$\underline{\underline{\therefore}}$ $P(B' \cap C') = 0.669$

(d) $P(B \cup C)$ = A^+ in at least 1 of two courses, either Biology (B) or chemistry (C)

$P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$= \frac{375}{1150} + \frac{50}{1150} - \frac{45}{1150}$

$= \frac{380}{1150} = 0.330$

$\underline{\underline{\therefore}}$ $P(B \cup C) = 0.330$

(c) $P(B' \cup C')$ = no A^+ in chemistry (C) or no A^+ in Biology (B), but not no A^+ in both (part (e))

complement of $P(B \cap C) = P(B' \cup C')$, $\therefore P(B' \cup C') = 1 - P(B \cap C)$

$\therefore P(B \cap C) = \frac{45}{1150}$

$P(B' \cup C') = 1 - \frac{45}{1150}$

$= \frac{1150 - 45}{1150}$

$= \frac{1105}{1150} = 0.961$

$\underline{\underline{\therefore}}$ $P(B' \cup C') = 0.961$

3.6) G: Y = yes answer

B = responder has bullied

~~D = rolling 1~~

D' = rolling 2-6

(a)

$P(B) = 0.13$

$P(D) = \frac{1}{6}$

$P(D') = \frac{5}{6}$

want to find $P(Y)$, knowing that $P(B) = 0.13$.

$\therefore P(Y|D) = P(B) = 0.13$

$P(Y|D') = P(B') = 1 - P(B)$

$= 1 - 0.13$

$= 0.87$

$\therefore P(Y) = P(Y|D)P(D) + P(Y|D')P(D')$

$= (0.13)(\frac{1}{6}) + (0.87)(\frac{5}{6})$

$= 0.022 + 0.725$

$P(Y) = 0.747$

(b) $P(Y) = P(Y|D)P(D) + P(Y|D')P(D')$

we now know that p = Probability of bullying a student, so

$\therefore p = P(B) = P(Y|D)$

Then, we can say

$P(Y) = p \cdot P(D) + p' \cdot P(D')$

$P(Y) = p \cdot (\frac{1}{6}) + (1-p) \cdot (\frac{5}{6})$

$\therefore P(Y) = p(\frac{1}{6}) + (1-p)(\frac{5}{6})$

$P(Y) = \frac{1}{6}p + \frac{5}{6} - \frac{5}{6}p$

$P(Y) - \frac{5}{6} = \frac{1}{6}p - \frac{5}{6}p$

$\frac{P(Y) - \frac{5}{6}}{-\frac{4}{6}} = \frac{-\frac{4}{6}p}{-\frac{4}{6}}$

$\therefore \frac{P(Y) - \frac{5}{6}}{-\frac{4}{6}} = p$

(c) knowing that $p = \frac{P(Y) - \frac{5}{6}}{-\frac{4}{6}}$ and now being given $P(Y) = \frac{275}{395}$,

we can say that

$\therefore p = \frac{\frac{275}{395} - \frac{5}{6}}{-\frac{4}{6}}$

$= \frac{0.696 - 0.833}{-0.667}$

$= \frac{-0.137}{-0.667}$

$p = 0.205$

$\therefore$ The probability (p) that a student has bullied a classmate is $p = 0.205$, or ~20.5%.

6.2) G: A = memory loss

B = difficulty completing tasks

$A \cap B$ = both symptoms.

(a) $P(A \cup B)$ = Probability that patient shows either A or B

$P(A) = 0.85$

$P(B) = 0.78$

$P(A \cap B) = 0.67$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.85 + 0.78 - 0.67$

$= 1.63 - 0.67$

$= 0.96$

$\therefore P(A \cup B) = 0.96$, or 96% chance that the patient shows either A or B symptoms.

(b) $P(A' \cup B')$: Probability that patient shows neither A or B

$P(A' \cup B') = 1 - P(A \cup B)$

$= 1 - 0.96$

$= 0.04$

$\therefore P(A' \cup B') = 0.04$, or 4% chance that a patient shows neither A or B symptoms.

6.6) G: 25 total flies

5 = black

20 = grey

(a) Sample of 2 flies, ∴ we can say that

A_i, where $i = 1, 2$

∴ A_1 = First selected fly

A_2 = Second selected fly

Probability that A_1 is black = $\frac{5}{25}$

∴ $P(A_2|A_1) = \frac{4}{24}$ = conditional probability A_2 is black.

unconditional probability A_2 is black = $P(A_2)$

$P(A_2) = P(A_2 \cap A_1) + P(A_2 \cap A_1')$

$= P(A_2|A_1)\,P(A_1) + P(A_2|A_1')\,P(A_1')$

we know that:

$P(A_1) = \frac{5}{25}$

$P(A_2|A_1) = \frac{4}{24}$

∴ $P(A_1') = 1 - \frac{5}{25}$

$= \frac{20}{25}$

$P(A_2|A_1') = \frac{5}{24}$

∴ $P(A_2) = \left(\frac{4}{24}\right)\left(\frac{5}{25}\right) + \left(\frac{5}{24}\right)\left(\frac{20}{25}\right)$

$= \frac{20}{600} + \frac{100}{600}$

$= \frac{120}{600} = \frac{1}{5}$

$= 0.20$

∴ $P(A_2|A_1) = \frac{4}{24} \neq \frac{1}{5} = P(A_2)$, so the events A_2 (second fly is black) is not independent of A_1 (first fly is black).

(b) G: 10,000 total flies

2000 = black

8000 = grey

A sample of 2 flies, ∴ A_i, where $i = 1, 2$

A_1 = first selected fly

A_2 = Second selected fly

probability that A_1 is black = $\frac{2000}{10\,000}$

∴ $P(A_2|A_1) = \frac{1999}{9\,999}$

$P(A_2) = P(A_2|A_1)\,P(A_1) + P(A_2|A_1')\,P(A_1')$

we know that:

$P(A_1) = \frac{2000}{10000}$

$P(A_2|A_1) = \frac{1999}{9999}$

∴ $P(A_1') = 1 - \frac{2000}{10000}$

$= \frac{8000}{10\,000}$

$P(A_2|A_1') = \frac{2000}{9999}$

∴ $P(A_2) = \left(\frac{1999}{9999}\right)\left(\frac{2000}{10000}\right) + \left(\frac{2000}{9999}\right)\left(\frac{8000}{10\,000}\right)$

$= (0.1999)(0.20) + (0.20002)(0.80)$

$= 0.0399 + 0.160016$

$= 0.1999$

∴ $P(A_2|A_1) = \frac{2000}{10000} = 0.20$

$P(A_2) = 0.1999 = \sim 0.20$

Hence, $P(A_2|A_1) = P(A_2)$, and the events are independent of each other.