MAT3172 Test 1 (September 22, 2021)

1. You have four keys; two of them open the door, and the other two don't. You try them one by one randomly (discarding the ones that do not fit) until finally you open the door on the X-th try. Find:

- the probability mass function of the random variable X;
- the expectation of X
- the variance of X.

2. Three fair coins are tossed. Describe the corresponding probability space. Let A be the event that the outcomes of the first and of the last tosses are different. Find the unconditional probability of A and its conditional probability under each of the conditions $B_i = \{\text{exactly } i \text{ coins are heads}\}$ for $i = 0, 1, 2, 3$.

3. A fair coin is tossed three times. Let X be the number of heads among the first two tosses and Y be the number of tails among the last two tosses. Find:

- the joint probability mass function of X and Y;
- the expectation of XY.

4. The joint density function of continuous random variables X and Y is

$$f_{XY}(x,y) = \begin{cases} C(x+y)\,, & 0 \le x \le y \le 1 \\ 0\,, & \text{otherwise} \end{cases}$$

Find:

- the constant C;
- the marginal densities f_X and f_Y;
- the expectation of $Y - X$.

5. X and Y are random variables with the joint density function

$$f_{XY}(x,y) = \begin{cases} Cxy\,, & 0 \le y \le x \le 2 \\ 0\,, & \text{otherwise} \end{cases}$$

Find:

- the constant C;
- the probability $P\{X + Y > 2\}$.