MAT1330 Study Group #7

November 9, 2022

Mentor: Amal Abed

aabed@uottawa.ca

Optimization: Tips

- Draw a picture. Label and identify variables (that which you can change) and parameters (values you are given), and the relations between them.
- Identifying the variable y you want to optimize, and find an equation for y in terms of JUST ONE other variable x. Then you can use Calculus to optimize y with respect to x.
- Interpret your results in words. Decide if they make sense!

1. Assume that snails in an escargot farm grow logistically and are harvested according to the following DTDS:

$$\boxed{x_{t+1} = 2.5x_t(1-x_t) - hx_t}$$

- The time step "t" is measured in months
- The quantity x_t is the population of snails at time t in units of thousands
- The constant parameter h > 0 represents the harvesting intensity (h=ratio of #snails harvested / #snails at start of month)

Goal: optimize harvest intensity to maximize long-term yield

a. What is the updating function?
b. What are the fixed points?
c. What are <u>assumptions</u> we should make regarding h? (hint: biologically relevant domain)
d. What is the mathematical <u>expression</u> for the long-term yield?
e. What is the <u>optimal</u> harvesting intensity (h) for this crop?
f. What is the corresponding <u>maximum</u> long-term yield?

a.) $f(x) = 2.5x(1-x) - hx$

b.) $f(x) = x \rightsquigarrow$ $x = 2.5x(1-x) - hx$

$0 = 2.5x - 2.5x^2 - hx - x$

$0 = 1.5x - hx - 2.5x^2$

$0 = x(1.5 - h - 2.5x)$

$\boxed{x^* = 0}$

ok cool, but not useful for solving

$0 = 1.5 - h - 2.5x$

$x^* = \dfrac{1.5-h}{2.5} = \boxed{0.6 - 0.4h}$

what we care about b/c "h"

c.) We want biologically relevant equilibria, so...

assumption 1 → $x^* = 0.6 - 0.4h \geq 0$

$0.6 \geq 0.4h$

$\boxed{h \leq 1.5}$ } domain is $[0, 1.5]$

assumption 2 → $\boxed{h \geq 0}$

d.) Formula to know for DTDS "long term yield": $H(h) = hx^*$

$$H(h) = hx^* = h(0.6 - 0.4h) = \boxed{0.6h - 0.4h^2}$$

e) Looking for critical point of H(h):

① $H(h) = 0.6h - 0.4h^2$

$H'(h) = 0.6 - 0.8h$

② $0 = 0.6 - 0.8h$

$\boxed{h = 0.75}$ ⇒ harvesting intensity 75%

f.) Max long-term yield = max "H"

☆ Is $h = 0.75$ the global max? ⇒ use EVT!

⇒ recall domain is $[0, 1.5]$

end pt ① $H(0) = 0.6(0) - 0.4(0)^2 = 0$

critical point ② $H(0.75) = 0.6(0.75) - 0.4(0.75)^2 = \boxed{0.225}$

end pt ③ $H(1.5) = 0.6(1.5) - 0.4(1.5)^2 = 0$

∴ by EVT test, $h = 0.75$ gives global max value of $H = 0.225$ ⇒ 225 snails/month

recall units are 1000 snails

2. Find the following limit:

$$\lim_{x\to 0^+}\left(\frac{1}{x}\right)^x$$

① Plug in limit: $\left(\frac{1}{0}\right)^0 = \infty^0$ ⇝ indeterminate in L'Hopital format!

② convert: $\left(\frac{1}{x}\right)^x = e^{x\ln(1/x)}$

③ Limit of exponent: $\lim_{x\to 0^+} x\ln(1/x) = \lim_{x\to 0^+} \frac{\ln(1/x)}{x^{-1}}$

L'H $\frac{\infty}{\infty}$ $\nearrow$ $= \lim_{x\to 0^+} \frac{\underbrace{\frac{1}{(1/x)}}_{=x}\cdot \cancel{-x^{-2}}}{\cancel{-x^{-2}}}$

$= \lim_{x\to 0^+} x = 0$

④ Whole limit: $\lim_{x\to 0^+}\left(\frac{1}{x}\right)^x = \lim_{x\to 0^+} e^{x\ln(1/x)} = e^0 = \boxed{1}$

3. Which of the following give indeterminate forms as “x goes to infinity”?

a. $\frac{e^x}{x}$ → ∞/∞ = ?
b. xe^{-x} → $\infty\cdot 0$ = ?
c. $\frac{e^{-x}}{x}$ → $0/\infty = 0$ ✓
d. $e^x - e^{2x}$ → $\infty - \infty$ = ?
e. $\frac{\ln(x)}{e^x}$ → ∞/∞ = ?
f. $\ln(x) + e^x$ → $\infty + \infty = \infty$ ✓

* Key here is knowing infinity rules & e^x / $\ln(x)$ graphs

↳ TRICK: if in doubt, plug BIG # for "∞" in calculator