ALGEBRA of FUNCTIONS - COMBINING FUNCTIONS

The ALGEBRA of functions focuses on building a new function by $+, -, \cdot, /$ or "composing" functions. By "composing" we mean one function becomes the input (independent variable) for the function, while the other function is the dependent variable variable (output).

ADDING and SUBTRACTING

Given $y = f(x)$ and $y = g(x)$, then we can "combine" these functions to construct new functions $y = (f \pm g)x$ where

$$(f+g)x = f(x) + g(x)$$
$$(f-g)x = f(x) - g(x)$$

$$D_{f \pm g} = D_f \cap D_g$$

INTERSECTION (Common to BOTH f and g).

Eg. ① Given $A = \{(1,3), (2,-5), (4,4), (-8,-1), (7,0)\}$

and $B = \{(4,7), (7,-1), (-1,8), (2,-3), (1,1)\}$

find the following, if possible.

SOLUTION:

i) D_A, D_B and D_{A+B}

$\therefore D_A = \{1, 2, 4, -8, 7\}$

$\therefore D_B = \{4, 7, -1, 2, 1\}$

$D_{A+B} = D_A \cap D_B$

$\therefore D_{A+B} = \{1, 2, 4, 7\}$

ii) $A+B$

1+3 and −5+(−3)

$\therefore A+B = \{(1,4), (2,-8), (4,11), (7,-1)\}$

Eg. ② Given $A=\{(1,3),(2,-5),(4,4),(-8,-1),(7,0)\}$

and $B=\{(4,7),(7,-1),(-1,8),(2,-3),(1,1)\}$

find the following, if possible.

SOLUTION:

i) D_A, D_B and D_{A-B}

$\therefore D_A=\{1,2,4,-8,7\}$

$\therefore D_B=\{4,7,-1,2,1\}$

$D_{A-B}=D_A\cap D_B$

$\therefore D_{A-B}=\{1,2,4,7\}$

ii) $A-B$

3-1 , -5-(-3)

$\therefore A-B=\{(1,2),(2,-2),(4,-3),(7,1)\}$

Eg. ③ Consider $f(x)=\frac{x+1}{x-1}$, $g(x)=\frac{x}{x+1}$ and $m(x)=x-3$.

Find the following, if possible.

i) $(f+g)(2)$

$= f(2)+g(2)$

$= 3+\frac{2}{3}$

$= \frac{11}{3}$

ii) $(g-m)(-3)$

$= g(-3)-m(-3)$

$= \frac{3}{2}+6$

$= \frac{15}{2}$

iii) $(f+g)(x)$

$= f(x)+g(x)$

$= \frac{x+1}{x-1}+\frac{x}{x+1}$

$= \frac{x^2+2x+1+x^2-x}{(x-1)(x+1)}$

$= \frac{2x^2+x+1}{(x-1)(x+1)}$

iv) D_{f-g}

$= D_f \cap D_g$

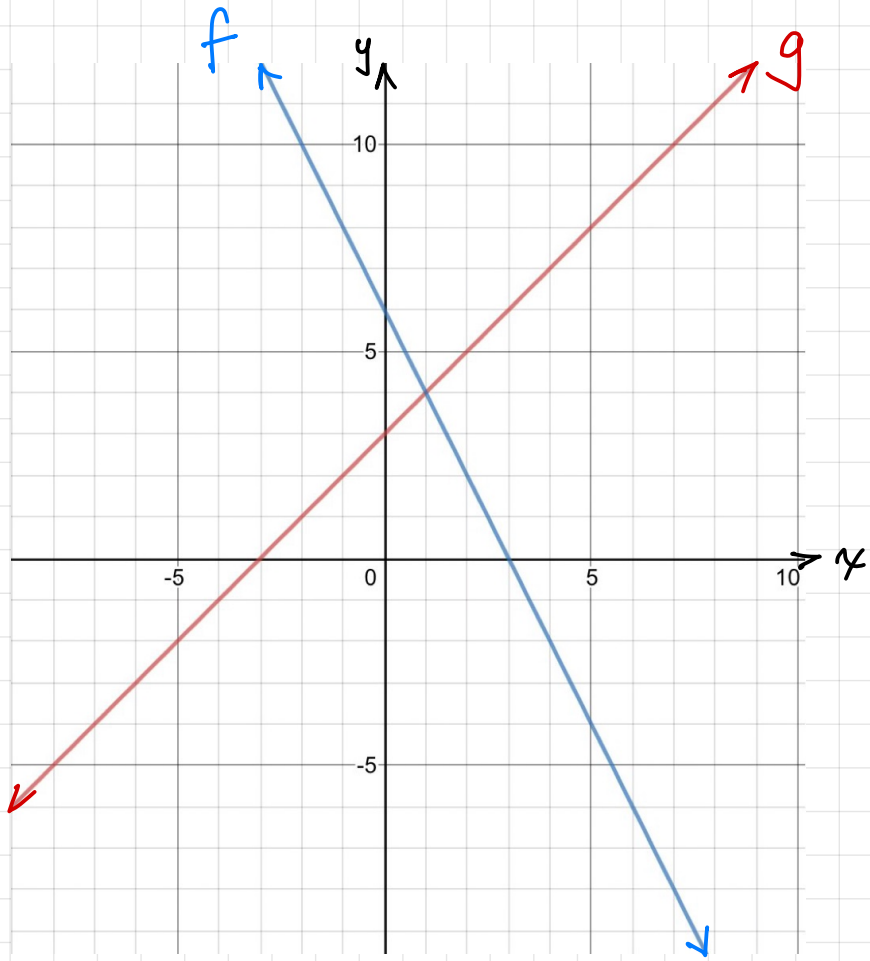
$= \{x \in \mathbb{R} \mid x \neq \pm 1\}$

v) $(g+m)(-1)$

DNE

$D_{g+m} = \{x \in \mathbb{R} \mid x \neq -1\}$

Eg. (4) Use the graphs of the two linear functions to sketch both $(f+g)(x)$ and $(f-g)(x)$ and then find the equations of these new functions.



Assigned Questions

Algebra with Functions and Composition

1. If $f(x) = 4x^2 + 3x + 2$ and $g(x) = 2x^2 - 5x - 6$ find $f+g$, $f-g$, fg, and f/g

2. Let $f(x) = 4x - 3$, $g(x) = 4x^2 - 7x + 3$ and $h(x) = x - 1$.

   Find $(f + g)(2)$, $(fh)(-1)$, $(fg)(0)$ and $(g/f)(5)$

3. Let $f(x) = 4x - 3$, $g(x) = 4x^2 - 7x + 3$ and $h(x) = x - 1$.

   Find $f + g$, fh, fg, and g/f.

TEXTBOOK QUESTIONS

1. Adding & Subtracting Functions

   Pg. 528-530 # 1-7, 11, 13, 16.

2. Multiplying & Dividing Functions

   Pg. 537-539 # 1-5, 7, 8, 10, 13, 17.

   Pg. 542 # 1 ade, 2.

3. Composite Functions

   Pg. 552 # 1-3, 5, 6.