Arithmetic Sequence

$a_n = a_1 + (n-1)d$

$S_n = \left(\frac{a_1+a_2}{2}\right) n$

Geometric sequence

$a_n = a_1 (r)^{n-1}$

$S_n = \frac{a_1(1-r^n)}{1-r}$

*Divergence Test

if $\lim_{n\to\infty} a_n \neq 0$, $\sum_{n=1}^{\infty} a_n \to$ diverges

if $\lim_{n\to\infty} a_n = 0 \to$ converges OR diverges

so do another test.

$\sum_{n=1}^{\infty} ar^{n-1}$, $|r|<1 \Rightarrow$ converges

$|r| \geq \Rightarrow$ diverges

*P-series test

$\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p>1 \Rightarrow$ converges

$p \leq 1 \Rightarrow$ diverges

*Telescoping series

$\sum a_n = 1 - \cancel{\tfrac{1}{2}} + \cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{3}} + \cancel{\tfrac{1}{3}} + \dots$ $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n = L \to$ converges

$= \pm\infty \to$ diverges

*Integral Test

$a_n = f(n)$, $\Rightarrow \int_1^{\infty} f(x)dx = L \to$ converges

$= \pm\infty \to$ diverges

*Ratio Test

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$ if $<1 \to$ converges

>1 or $\infty \to$ diverges

$=1 \to$ inconclusive

*Root test

$\lim_{n\to\infty} \sqrt[n]{|a_n|}$ $<1 \to$ converges

$>1 \to$ diverges

$=(|a_n|)^{\frac{1}{n}} = 1 \to$ inconclusive

*Direct Comparison Test

$0 \leq a_n \leq b_n$

If $\sum b_n$ converges, $\sum a_n$ converges

If $\sum b_n$ diverges, $\sum a_n$ diverges

*Limit comparison Test

$\lim_{n\to\infty} \frac{a_n}{b_n} = L$ if $\sum a_n$ converges, $\sum b_n$ converges

$\sum a_n$ diverges, $\sum b_n$ diverges

*Absolute value test

If $\sum|a_n|$ converges, then $\sum a_n$ converges $\Rightarrow \nabla$ Absolutely convergent

$\to$ If $\sum|a_n|$ diverges and $\sum a_n$ converges $\Rightarrow \nabla$ conditionnally convergent

$\to$ if $\sum|a_n|$ diverges and $\sum a_n$ diverges $\Rightarrow$ divergent

*Alternating Series Test

$\sum_{n=1}^{\infty} (-1)^n a_n \to$ for converge: both conditions must apply

(1) $\lim_{n\to\infty} a_n = 0$ and (2) $a_{n+1} \leq a_n$

*Cauchy Criteria ⇒ basically root test

$\sum_{n=n_0}^{\infty} a_n \to$ (1) If $\lim_{n\to\infty} (|a_n|)^{1/n} = L < 1 \Rightarrow$ absolument convergente

(2) Si $\lim_{n\to\infty} (|a_n|)^{1/n} = L > 1 \Rightarrow$ divergent

(3) If $\lim_{n\to\infty} (|a_n|)^{1/n} = 1 \Rightarrow$ inconclusive

*For alternating series

$|\text{error}| = |S - S_n| \leq b_{n+1}$

ex: S_{1000}, $a_n = \frac{n}{n^2+4}$

$b_{1001} = \frac{1001}{(1001)^2+4} \approx 0.0009$

*D'Alembert Criterion ⇒ basically ratio test

$\to$ If $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} < 1 \Rightarrow$ converges absolutely

$\to$ If $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} > 1 \Rightarrow$ diverges

*Power Series

$\sum_{n=0}^{\infty} c_n(x-a)^n$ & R = radius of convergence

$\to$ If $|x-a| < R \to$ converges

$\to$ If $|x-a| > R \to$ diverges

$a-R < x < a+R \to$ converges

$x < a-R$ and $x > a+R \to$ diverges