

MULTIPLE-CHOICE QUESTIONS Questions 1–4 are **multiple-choice questions** worth 2 points each. Your answers to multiple-choice questions do not need to be justified. When you reach your answer, clearly indicate the question number and write the letter of your response beside the question number:

For example: (*write out your scrap work, but it will not be graded*)

(*clearly indicate your final choice*) **Q1.** [letter of your choice]

Q1. The mass M (in grams) of a radioactive sample is reduced to 75 % of its initial mass after 4 years. Assume M satisfies the differential equation $\frac{dM}{dt} = kM$, where t is measured in years. If the mass is initially 10 g, find the mass (in grams) that would remain after 9 years. Round your answer to 1 decimal place.

- A. 8.2 B. 6.3 C. 4.9 D. 7.1 E. 3.4 F. 2.6 G. 5.2

Q2. Which one of the following statements is always true?

A. If the sequence $\{a_n\}_{n=1}^{\infty}$ converges to 0, then the series $\sum_{n=1}^{\infty} a_n$ must converge.

B. If the sequence $\{a_n\}_{n=1}^{\infty}$ converges, then the series $\sum_{n=1}^{\infty} a_n$ must converge.

C. If the series $\sum_{n=1}^{\infty} a_n$ diverges, then the sequence $\{a_n\}_{n=1}^{\infty}$ must diverge.

D. If the sequence $\{a_n\}_{n=1}^{\infty}$ diverges, then the series $\sum_{n=1}^{\infty} a_n$ must diverge.

Q3. If it is convergent, find the sum of the geometric series $\sum_{m=1}^{\infty} \frac{5 \cdot (-2)^{m+1}}{3^{2m-1}}$.

- A. $\frac{60}{11}$ B. $-\frac{60}{11}$ C. $\frac{60}{7}$ D. $-\frac{60}{7}$ E. $\frac{45}{11}$ F. $\frac{45}{7}$ G. It diverges.

Q4. Fact: the series $S = \sum_{m=1}^{\infty} \frac{(-1)^m}{m^{5/2}}$ converges by virtue of the Alternating Series Test.

According to the Alternating Series Estimation Theorem, what is the smallest value of N for which the error $|R_N| = |S - S_N|$ is at most 0.005 ?

- A. $N = 12$ B. $N = 11$ C. $N = 9$ D. $N = 10$ E. $N = 7$ F. $N = 8$

LONG-ANSWER QUESTIONS Questions 5–7 are **long-answer questions** worth a total of 17 points. For long-answer questions, **all of your work must be justified and your steps must be written in a clear and logical order**. Clearly indicate Question numbers.

For example: **Q5(a).** [write a fully justified solution].

Q5. [6 points] For each integer $k \geq 1$, let $a_k = \ln(2k + 3) - \ln(2k + 1)$.

a) Consider the **sequence** $\{a_k\}_{k=1}^{\infty} = \{\ln(2k + 3) - \ln(2k + 1)\}_{k=1}^{\infty}$.

Is this sequence convergent or divergent? If it converges, find its exact value. Show all your work to justify your answer. Be sure to use appropriate mathematical notation.

b) Consider the **telescopic series** $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} (\ln(2k + 3) - \ln(2k + 1))$.

Find a simplified expression for the n th partial sum S_n , by cancelling terms telescopically. Show all your work!

c) Determine whether the series $\sum_{k=1}^{\infty} a_k$ converges or diverges. If it converges, find its exact sum; otherwise, briefly explain why it diverges.

Q6. [6 points] Use appropriate series tests to determine whether each of the following series is convergent or divergent.

If you apply a certain test, name it, and state and show your work to verify the conditions needed in order for the test to apply.

Show all your work. Clearly explain your conclusion.

(a)
$$\sum_{n=1}^{\infty} \frac{9 \sin(n)}{2^n}$$

(b)
$$\sum_{N=1}^{\infty} N e^{-N^2}$$

Q7. [5 points] Consider the power series:
$$\sum_{n=0}^{\infty} \frac{(x + 1)^n}{2^{2n}(n^2 + 7)}$$

(a) Find its radius of convergence. Show your work.

(b) Find its interval of convergence. Show your work.

end of exam!