ECON 2020 B Final Examination

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Instructions: Answer all questions in the space provided. Show your work for each one. **66 pts**

1. Consider the production function given by **11 pts**

$$q = \left(1 + L^{-1}K^{-1}\right)^{-1}.$$

(a) Determine the marginal products of labour and capital. *1.5 pts*

$$q = (1 + L^{-1}K^{-1})^{-1} \qquad q = \left(\frac{LK+1}{LK}\right)^{-1}$$

$$= \left(1 + \frac{1}{LK}\right)^{-1} \qquad q = \frac{KL}{1+KL}$$

(b) Determine the marginal rate of technical substitution. *1 pt*

$$MRS = -\frac{MP_L}{MP_K} = \frac{L/(1+LK)^2}{K/(1+LK)^2} = \frac{L}{K}$$

(c) Show that the given production function exhibits diminishing marginal products. *3 pts*

$$\frac{\partial MP_K}{\partial K} = \frac{-L}{(L+LK)^4} \cdot 2(1+LK) \cdot L < 0$$

$$\frac{\partial MP_K}{\partial L} = \frac{-K}{(1+LK)^4} \cdot 2(1+LK) \cdot K < 0$$

Diminishing marginal product because they are both less than 0

Both are

(d) Taking an appropriate limit, find the upper bound on q for the given production function. *1.5 pts*

In such situation $q = \frac{LK}{1+LK} < 1$ for all values of L, K

Therefore, 1 is the upper bound for the function q

question 1a continuation

a) Marginal product of labour $(MP_L) = \frac{dQ}{dL}$

$$= \frac{d}{dL}\left(\frac{LK}{LK+1}\right)$$

$$= \frac{(1+LK)(K) - (KL)(K)}{(1+LK)^2}$$

$$= \frac{K + \cancel{LK^2} - \cancel{LK^2}}{(1+LK)^2}$$

$$MP_L = K/(1+LK)^2$$

$$\frac{dQ}{dK} = \frac{d}{dK}\left(\frac{LK}{LK+1}\right)$$

$$= \frac{L + \cancel{L^2K} - \cancel{L^2K}}{(1+LK)^2}$$

$$MP_K = L/(1+LK)^2$$

(e) A *local* measure of the returns to scale incorporated in a production function is given by the *scale elasticity* defined as 2 pts

$$e_q(L, K) = \left. \frac{\partial f(xL, xK)}{\partial x} \frac{x}{q} \right|_{x=1},$$

where $\cdot|_{x=1}$ means evaluate the preceding expression at $x = 1$ after working it out. Determine this elasticity for the given production function and use it to show that $e_q(L, K) \gtreqless 1$ for $q \lesseqgtr \frac{1}{2}$.

(f) Determine the average product of labour and show that it is a decreasing function of labour employed. 2 pts

2. Suppose that the VHS-videotape-transfer industry is comprised of a large number of identical firms each of which can digitize 5 tapes per day at an average cost of \$10 per tape. A royalty must also be paid by each firm to film studios at a per-film/tape rate r that is an increasing function of total industry output Q: 11 pts

$$r = 0.002Q\,.$$

Market demand for tape digitizations is given as

$$Q = 1\,050 - 50p\,. \quad \text{—(i)}$$

(a) Assuming the industry is in long-run equilibrium, what will be the market price and quantity of tape digitizations? How many tape-digitization firms will there be? What will the per-film/tape royalty rate be? 3 pts

MC = 10
MC = production + royalty
= 10.002
P = 10.002 at L.R eq.
Q = 1050 − 50×10×10.002
Q = 549.9
549.9 = 1050 − 50P
50P = 500.1
P = 10
There will be ≈ 110 tape digitising firms

r = 0.002Q
r = 0.002(549.9)
r ≈ \$1.1

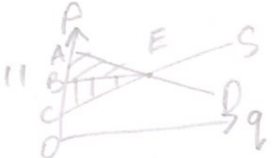


Fig d i

$500P - 7750 = 0$

Now suppose that the government institutes a \$5.50 per-firm/tape tax on the tape-digitization industry.

(b) How will this tax affect the market equilibrium? In other words, what will be the new equilibrium price, quantity, number of firms, and royalty rate? *3 pts*

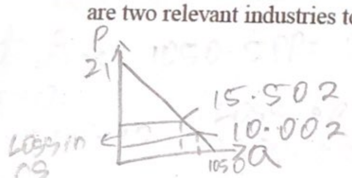
$P = 5.5 + 10.002$
$= 15.502$
$Q = 1050 - 50 \times 15.502$
$= 274.9$

$r = 0.002Q$
$= 0.002(274.9)$
$r = \$0.55$

(c) How will the burden of the tax be allocated between consumers and producers? *1 pt*

It will be collected from consumers

(d) What will be the loss of consumer and producer surplus due to the tax? (Hint: There are two relevant industries to consider in relation to the latter.) *3 pts*



$= 5.5 \times \frac{(549.9 + 274.9)}{2}$
$= 2268.2$

$\therefore$ Loss in CS $= 2268.02$
No Loss in PS because market is in perfect complebition

(e) What will be the deadweight loss of the tax? *1 pt*

No deadweight loss of tax

3. A single firm has a monopoly in the market for doodads and can produce at constant average and marginal costs of C dollars per doodad. The firm faces the market demand curve given by *12 pts*

$$Q = A - Bp,$$

where A, B, and C are positive parameters such that $A > BC$.

$= 50 - 2Q$

B

(a) Find the profit-maximizing quantity, price, and profit as functions of A, B, and C. *3 pts*

$TR = P(Q)Q = \left(\frac{A-Q}{B}\right)Q = \frac{AQ-Q^2}{B}$

$MR = \frac{\partial TR}{\partial Q} = \frac{A-2Q}{B}$

$MC = C$

$MR = MC$

$\frac{A-2Q}{B} = C \Rightarrow A-2Q = BC$

$\frac{A-BC}{2} = Q^*$

$P^* = \frac{A-Q}{B} \quad \frac{A - \frac{A-BC}{2}}{B} = (A+BC)/2B$

$P^* = \frac{A+BC}{2B}$

$\pi = (P-MC)Q = \left[\frac{A+BC}{2B} - C\right]\left[\frac{A-BC}{2}\right]$

$\pi^* = \frac{(A-BC)^2}{4B}$

(b) Use your part-(a) results to complete the following table: *3 pts*

(A,B,C)	Q	p
(60, 1, 10)	25	35
(25, 0.5, 10)	10	30
(80, 2, 10)	30	25

(c) Use your part-(b) results to explain why a monopoly has no supply curve in the sense that perfectly competitive firms or industries do. *2 pts*

A monolopy has no well defined curve because output decision depends on MC and demand function.

(d) Plot the demand and cost curves given in the part-(b) table along with the associated profit-maximizing quantity-price pairs in the following graph after labelling appropriately the two axes. *4 pts*

demand curve 1

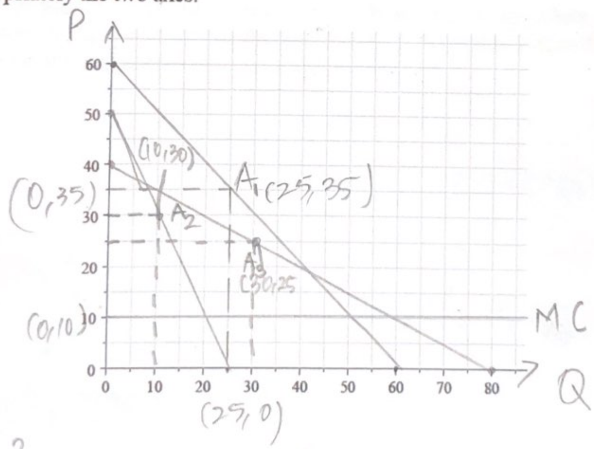
$A=60$

$B=1$

$C=10$

$P = \frac{A-Q}{B} = 60-Q$

$MC=10$



demand curve 2

$A=25$

$B=0.5$

$C=10$

$P = \frac{A-Q}{B} = \frac{25-Q}{0.5} = 50-2Q$

demand curve 3

$A=80$

$B=2$

$C=10$

$P = \frac{80-Q}{2} = 40 - \frac{Q}{2}$

4. Consider a monopoly market with a demand function for which quantity demanded depends not only on price p but also on the amount of advertising the firm does, A, measured in dollars as are other costs. The specific form of this function is 10 pts

$$Q = (20 - p)\, B(A)\,,$$

where

$$B(A) = 1 + 0.1A - 0.01A^2\,.$$

The firm's cost function is

$$C(Q, A) = 10Q + 15 + A\,.$$

(a) What is the firm's (unmaximized) profit function $\Pi(Q, A)$? *2 pts*

π = Revenue − Cost

$R = P \times Q$

$Q = (20-P)B(A)$

$\frac{Q}{B(A)} = 20 - P$

$P = 20 - \frac{Q}{B(A)}$

$R = (20 - Q/B(A)) \cdot Q$

$\pi = \cancel{20Q} - \frac{Q^2}{B(A)} - 10Q + 15 + A$

$\pi = 10Q - \frac{Q^2}{B(A)} - 15 - A$

$\pi = 10Q - \frac{Q^2}{1+0.1A-0.01A^2} - 15 - A$

(b) What are the first-order necessary conditions for a pair of values $Q > 0$ and $A > 0$ to maximize monopoly profit? *2 pts*

First order condition

$\frac{\partial \pi}{\partial Q} = 0$ and $\frac{\partial \pi}{\partial A} = 0$

$\frac{\partial \pi}{\partial Q} = 10 - \frac{2Q}{1+0.1A-0.01A^2} - A$ — (1)

$\frac{\partial \pi}{\partial A} = 10Q - \frac{Q^2 \cdot (0.1 - 0.02A)}{(1+0.1A-0.01A^2)^2} - 1 = 0$ — (2)

(c) Use your part-(b) conditions to solve for the profit-maximizing values of Q and A, and then use these to determine the market price p, the maximized profit level π, and the associated amount of consumer surplus. *4 pts*

From equation 1

$\frac{10 - 2Q}{1+0.1A-0.01A^2} = A$

$\Rightarrow 10 + A - 0.1A^2 - 2Q = A + 0.1A^2 - 0.01A^3$

$0.2A^2 - 0.01A^3 + 2Q = 10$

From equation 2

$10Q(1+0.1A-0.01A^2)^2 -$

$Q^2(0.1-0.02A) -$

$(1+0.1A) - 0.01A^2)^2 = 0$

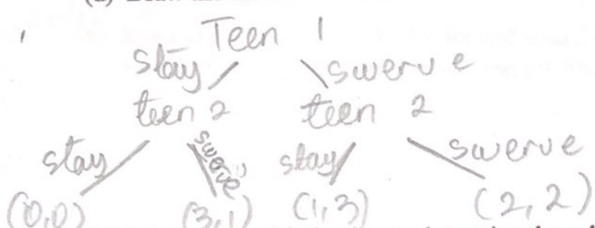
(c) Determine the players' expected payoffs in the part-(a) and part-(b) equilibria. *1.5 pts*

* Expected payoff for mixed-strategy nash equilibria is $= 2 \times 0.5 + 1 \times 0.5$
$= 1.5$

* 1) (Stay, Swerve) payoff of 3 to teen 1 and 1 to teen 2
2) (Swerve, stay) payoff of 1 to teen 1 and 3 to teen 2 } Pure strategy nash equilibria

Now suppose the game is played sequentially, with Teen 1 moving first and committing to their action (Swerve or Stay) by throwing away the steering wheel.

(d) Draw the extensive form for this version of the game. *2 pts*



(e) Use backward induction to determine the subgame-perfect equilibrium. *2 pts*

teen 1 stay, game is teen 2
stay (0,0) swerve (3,1)
teen 1 will swerve 1>0

when teen 1 swerves game is teen 2
stay (1,3) swerve (2,2)
teen 2 will 'stay' as 3>2

due to backward induct game becomes teen 1
stay (3,1) swerve (1,3)
teen 1 will 'stay' as 3>1

subgame perefect N.E is (stay, swerve)

For extra credit:

(f) Determine the pure-strategy Nash equilibria in terms of Teen 1's strategy and Teen 2's *contingent* strategies. *2 pts*

6. In a Stackelberg duopoly, one firm is a "leader" and the other is a "follower." Both firms know each other's costs as well as market demand. The follower acts like a Cournot competitor; the leader takes the follower's best-response function as given and picks its own output accordingly. Suppose that firms 1 and 2 face market (inverse) demand 11 pts

$$p = 100 - (q_1 + q_2)$$

and have costs $C_1 = 10q_1$ and $C_2 = q_2^2$, respectively.

$P = 100 - (q_1 + q_2)$

$C_1 = 10q_1 \quad C_2 = q_2^2$

$MC_1 = 10 \quad MC_2 = 2q_2$

finish writing at the back

(a) Determine each firm's best response as a function of the other firm's assumed level of output. *3 pts*

Firm 1) Profit $\pi_1 = TR_1 - C_1$

$TR_1 = (100 - q_1 - q_2)q_1$

$= 100q_1 - q_1^2 - q_1 q_2$

$MR = \frac{\partial TR_1}{\partial q_1} = 100 - 2q_1 - q_2$

π max condition $MR_1 = MC_1$

$100 - 2q_1 - q_2 = 10$

$\frac{90 - q_2}{2} = q_1$ or

$q_1 = 45 - \frac{q_2}{2}$

(b) Assuming that firm 1 is the leader and firm 2 the follower, determine the Stackelberg equilibrium outputs, market price, and profits. *3.5 pts*

If firm 1 is leader

$TR_1 = (100 - 25 + \frac{q_1}{4} - q_1)q_1$

$= (75 - \frac{3q_1}{4})q_1$

$MR = \partial TR / \partial q_1 = 75 - \frac{3q_1}{2}$

$MC = 10$

Profit max $MR_1 = MC_1$

$75 - \frac{3q_1}{2} = 10$

$65 = \frac{3q_1}{2}$

$q_1 \simeq 43$

from (ii) $q_2 = 25 - 43/4 \simeq 14$

$\therefore P = 100 - 43 - 14 = > 43$

π for firm 1 $= 43 \times 43 - 10 \times 43$

$= 1419$

π for firm 2 $= 43 \times 14 - (14)^2$

$= 406$

finish

(c) Assuming instead that firm 2 is the leader and firm 1 the follower, determine the Stackelberg equilibrium outputs, market price, and profits. *3.5 pts*

$TR_2 = (100 - 45 + \frac{q_2}{2} - q_2)q_2$

$= (55 - \frac{q_2}{2})q_2$

$MR_2 = \frac{\partial TR_2}{\partial q_2} = 55 - q_2$

$MC_2 = 2q_2$

π max $MR_2 = MC_2$

$55 = 3q_2$

$q_2 \simeq 18$

from (i) $q_1 = 45 - \frac{q_2}{2} \simeq 36$

$\therefore P = 100 - 13 - 36$

$= 46$

finish

(d) Given the foregoing, would firm 1 prefer to be the leader or the follower? What about firm 2? *1 pt*

Firm 1 would prefer to be leader because when its in the position of being leader is produces about 43 units but produces about 36 units when following. It wou

finish