Version A

SOLUTIONS

Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

Discrete Mathematics for Computing — Instructor: Elizabeth Maltais

MAT1348C — Test 1 — Tuesday, January 24, 2017

Instructions

- Clearly write your name and student number on this test, and **sign it** below to confirm that you have read, understand, and will follow these **instructions**:
- This is a 75-minute **closed-book** test. No notes are allowed. No calculators are allowed.
- **Put away everything** except a few pens or pencils, an eraser, and your student id. card.
- The exam consists of 9 questions on 9 pages. Page 9 contains a **Table of Logical Equivalences** to use in Question 9.
- Questions 1–4 are **short-answer**. Write the final answer in the appropriate answer box. You do not need to show any other work.
- Questions 5–6 are **true–false**. In each part, you must circle the correct response. You do not need to justify your answers.
- Questions 7–9 are **long-answer**. To receive full marks, your solution/proof must be complete, correct, and show all relevant details.
- Read all questions carefully and be sure to follow the instructions for the individual problems. You may ask for clarification.
- For rough work or additional work space, you may use the backs of pages. **Do not use any of your own scrap paper**.
- You must use **proper mathematical notation and terminology**.

Cellular phones, unauthorized electronic devices, or course notes are not allowed during this test. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession such as in your pockets. If caught with such a device or document, academic fraud allegations may be filed which may result in you obtaining zero for this test.

† **By signing below, you acknowledge that you have read, understand, and will comply with the above instructions.**

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Do not write in this table.

Question	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Total
Maximum points	2 pts	2 pts	3 pts	6 pts	3 pts	4 pts	5 pts	5 pts	6 pts	36 points
Marks obtained										

Short-answer Questions.

Write your final answer in the answer box. No justification is needed.

Q1. [2 points]

Let X be a compound proposition consisting of propositional variables p, q, and r. Suppose that X is true if and only if exactly one of p, q, and r is false.

Give a **disjunctive normal form** of X. *Do not simplify your answer.*

DNF: $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$

Q2. [2 points] Complete the following **definition**:

Propositions P and Q are called **logically equivalent** if

$P \leftrightarrow Q$ is a tautology.

Q3. [3 points]

Using only the propositional variable p and the logical connectives $\neg$ and $\wedge$, give an example of a compound proposition that is

i. a contradiction

ii. a tautology

iii. a contingency

Note. Each of your propositions must only use the variable p. Each must include **both** the logical connectives $\neg$ and $\wedge$ and **no other** logical connectives.

i. a contradiction: $p \wedge \neg p$

ii. a tautology: $\neg(p \wedge \neg p)$

(there are other possible answers)

iii. a contingency: $\neg p \wedge \neg p$

Q4. [6 points] Consider the following propositional variables:

F : "The missing skier is found."
W : "The missing skier is wounded."
R : "The rescue team returns safely."
S : "The rescue mission is successful."

Translate each of the following sentences into compound propositions using propositional variables F, W, R, and S. *Parentheses [] are included to clarify the structure.*

i. **The rescue mission is unsuccessful only if the missing skier is not found.**

i. *Compound proposition:* $\neg S \rightarrow \neg F$

ii. **In order for the rescue mission to be successful it is sufficient that [the missing skier is not wounded and the rescue team returns safely].**

ii. *Compound proposition:* $(\neg W \wedge R) \rightarrow S$

iii. **[The rescue mission is unsuccessful unless the missing skier is found] if and only if [the missing skier is wounded but the rescue team returns safely].**

iii. *Compound proposition:* $(\neg S \vee F) \leftrightarrow (W \wedge R)$

True–False Questions.

Circle the correct response, **T** (true) or **F** (false). No justification is needed.

Q5. [3 points]

Let X be the following compound proposition consisting of propositional variables $p_1, p_2, \ldots, p_n$, and c.

$$X: \quad (p_1 \vee p_2 \vee \cdots \vee p_n) \rightarrow c$$

Regarding the compound proposition X, determine whether each of the following statements is true or false:

(a) If at least one of $p_1, p_2, \ldots, p_n$ is true, then X will certainly be true. **T** **F**

(b) If c is true, then X will certainly be true. **T** **F**

(c) If $p_1, p_2, \ldots, p_n$ are all false, then X will certainly be false. **T** **F**

Q6. [4 points] Determine whether each of the following statements is true or false:

(a) If P and Q are any two contradictions, then $P \equiv Q$. **T** **F**

(b) If compound propositions P and Q are logically equivalent, then $P \oplus Q$ is a contradiction **T** **F**

(c) If A, B, C, and D are any propositions such that the set $\{A, B, C\}$ is inconsistent, then $(A \wedge B \wedge C) \rightarrow D$ is a tautology. **T** **F**

(d) If A, B, and C are any propositions such that the set $\{A, B, C\}$ is consistent, then $A \vee B \vee C$ is a tautology. **T** **F**

Long-Answer Questions.

Detailed solutions are required.

Q7. [5 points]

On The Island of Knights and Knaves, as you know, there are two types of inhabitants, indistinguishable by sight: knights, who always tell the truth, and knaves, who always lie. Strolling on The Island of Knights and Knaves, we meet two inhabitants A and B.

Using whichever method you prefer (the truth table method or by reasoning in words), answer the questions below. You must justify your claims with an explanation.

i. Person B says: "I am a knave if and only if A is a knave."

What do you conclude about the types (knight or knave) of persons A and B? Be as precise as possible.

(Truth Table Method)

Define I-am-a-Knight atoms: a: "A is a Knight."
b: "B is a Knight."

Translate: B says $\neg b \leftrightarrow \neg a$

a	b	$\neg b \leftrightarrow \neg a$	
T	T	T	← B is a Knight and his statement is T (possible)
T	F	F	← B is a Knave and his statement is F (possible)
F	T	F	← B is a Knight but his statement is F (impossible)
F	F	T	← B is a Knave but his statement is T (impossible).

These are the only possibilities. In both possible scenarios, A is a Knight.

∴ A is a Knight but we cannot determine what B is yet.

ii. Suppose, after person B speaks (as in part **i.**), person A then says "B is a knave but I am not." What do you conclude now?

(reasoning in words)

Given that A is a Knight (from part i) we know his statement must be True.

∴ B is a Knave and A is not a Knave must be True

∴ B must be a Knave (and A is still a Knight).

Q8. [5 points] Let X be the following compound proposition:

$$X: \quad \big((P \oplus Q) \wedge (P \vee R)\big) \rightarrow \big(Q \wedge R\big)$$

i. Construct a **complete truth table** for X, showing all relevant work.

ii. Is X a tautology, a contradiction, or a contingency? Write your answer in the box below.

i.

P Q R	P⊕Q	P∨R	(P⊕Q)∧(P∨R)	Q∧R	((P⊕Q)∧(P∨R))→(Q∧R)
T T T	F	T	F	T	T
T T F	F	T	F	F	T
T F T	T	T	T	F	F
T F F	T	T	T	F	F
F T T	T	T	T	T	T
F T F	T	F	F	F	T
F F T	F	T	F	F	T
F F F	F	F	F	F	T

(X)

ii. Proposition X is a contingency.

Q9. [6 points]

Let x and y be propositional variables. Prove each of the following **using only the equivalences listed on Page 9**. Justify each step by giving the name or number of the corresponding equivalence on Page 9. *Do not skip steps or combine several equivalences into a single step. Do not omit necessary parentheses.*

i. Prove the following: $y \wedge \neg(x \wedge y) \equiv \neg x \wedge y$

$y \wedge \neg(x \wedge y) \equiv y \wedge (\neg x \vee \neg y)$ (De Morgan's #19)

$\equiv (y \wedge \neg x) \vee (y \wedge \neg y)$ (Distributive #18)

$\equiv (y \wedge \neg x) \vee (F)$ (Negation #5)

$\equiv y \wedge \neg x$ (Identity #6)

$\equiv \neg x \wedge y$ (Commutative #14)

ii. Prove the following: $(x \rightarrow \neg y) \wedge (y \rightarrow x) \equiv \neg y$

$(x \rightarrow \neg y) \wedge (y \rightarrow x) \equiv (\neg x \vee \neg y) \wedge (y \rightarrow x)$ (Implication #1)

$\equiv (\neg x \vee \neg y) \wedge (\neg y \vee x)$ (Implication #1)

$\equiv (\neg y \vee \neg x) \wedge (\neg y \vee x)$ (Commutative #13)

$\equiv \neg y \vee (\neg x \wedge x)$ (Distributive #17)

$\equiv \neg y \vee (F)$ (Negation #5)

$\equiv \neg y$ (Identity #6)

Table of Logical Equivalences *You may detach this page.*

	Equivalence	Name
1.	$P \to Q \equiv \neg P \vee Q$	Implication Law
2.	$P \leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$	Biconditional Laws
3.	$P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$	
4.	$P \vee \neg P \equiv \mathbf{T}$	Negation Laws
5.	$P \wedge \neg P \equiv \mathbf{F}$	
6.	$P \vee \mathbf{F} \equiv P$	Identity Laws
7.	$P \wedge \mathbf{T} \equiv P$	
8.	$P \vee \mathbf{T} \equiv \mathbf{T}$	Domination Laws
9.	$P \wedge \mathbf{F} \equiv \mathbf{F}$	
10.	$P \vee P \equiv P$	Idempotent Laws
11.	$P \wedge P \equiv P$	
12.	$\neg\neg P \equiv P$	Double Negation Law
13.	$P \vee Q \equiv Q \vee P$	Commutative Laws
14.	$P \wedge Q \equiv Q \wedge P$	
15.	$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$	Associative Laws
16.	$(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$	
17.	$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$	Distributive Laws
18.	$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$	
19.	$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$	De Morgan's Laws
20.	$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$	