MULTIPLE-CHOICE QUESTIONS

Questions 1–5 are **multiple-choice questions** worth 2 points each. Your answers to multiple-choice questions do not need to be justified. You may write your scrap work on your paper but it will not be graded. When you reach your answer, clearly indicate the question number and write the letter of your response beside the question number:

For example: (*write out your scrap work, but it will not be graded*)

(*clearly indicate your final choice*) **Q1.** [`letter of your choice`]

Q1. Exactly two of the following improper integrals **converge**. Which 2 integrals below are **convergent**?

A. $\displaystyle\int_1^\infty \frac{\sin(x)+2}{x^{1/3}}\,dx$ **B.** $\displaystyle\int_1^\infty \frac{e^x-2}{3e^{4x}}\,dx$ **C.** $\displaystyle\int_0^1 \frac{2}{t}\,dt$

D. $\displaystyle\int_1^\infty \frac{x+1}{\sqrt{x}}\,dx$ **E.** $\displaystyle\int_0^1 \frac{1}{x^3}\,dx$ **F.** $\displaystyle\int_0^1 \frac{\sin^2(t)}{\sqrt{t}}\,dt$

Q2. Let R be the region enclosed by the curve $y = x^2$ and the line $y = 2x$.

Let S be the solid of revolution obtained by rotating R about the line $y = 4$.

Using the method of washers, determine which of the following definite integrals computes the total volume of S.

A. $\displaystyle 2\pi\int_0^2 (4-x^2)^2-(4-x)^2\,dx$ **B.** $\displaystyle 2\pi\int_0^2 x(2-x^2)dx$ **C.** $\displaystyle \pi\int_0^2 (4-x)(2-x^2)\,dx$

D. $\displaystyle \pi\int_0^2 \Big((4-x^2)-(4-x)\Big)^2 dx$ **E.** $\displaystyle \pi\int_0^2 (4-x^2)^2-(4-2x)^2\,dx$ **F.** $\displaystyle 2\pi\int_0^2 x(x^2-2)dx$

Q3. Which of the following integrals computes the **arc length** of the curve $y = e^{x^3}+\pi$ on the interval $[1,3]$?

A. $\displaystyle\int_1^3 \sqrt{1+9x^4e^{2x^3}}\,dx$ **B.** $\displaystyle\int_1^3 \sqrt{1+16x^4e^{x^3}}\,dx$ **C.** $\displaystyle\int_1^3 \sqrt{1+x^{-4}e^{2x^2}}\,dx$

D. $\displaystyle\int_1^3 \sqrt{1+x^4+e^{2x^3}}\,dx$ **E.** $\displaystyle\int_1^3 \sqrt{1+9x^4-e^{3x^3}}\,dx$ **F.** $\displaystyle\int_1^3 \sqrt{9+9x^4e^{x^3}}\,dx$

G. $\displaystyle\int_1^3 \sqrt{1-9+x^4e^{2x^3}}\,dx$ **H.** $\displaystyle\int_1^3 \sqrt{1-16x^3e^{x^3}}\,dx$ **I.** $\displaystyle\int_1^3 \sqrt{1+9x^3e^{2x^3}}\,dx$

Q4. Find the average value of $f(x) = 3x^2 \ln(5x)$ on the interval $1 \le x \le 4$. Round your answer to three decimal places.

A. 17.325 **B.** 45.645 **C.** 58.430 **D.** 31.649 **E.** 48.042 **F.** 41.974 **G.** 56.372 **H.** 20.105

Q5. Solve the initial value problem $\dfrac{dy}{dx} = 3\pi x^2 y$ where $y(0) = 12$.

A. $y = 12e^{\frac{\pi x}{3}}$ **B.** $y = 12e^{\pi x^3}$ **C.** $y = e^{12\pi x^3}$

D. $y = 12 + e^{\pi x^3}$ **E.** $y = 12e^{\frac{x^3}{\pi}}$ **F.** $y = 12 - e^{12\pi x^3}$

LONG-ANSWER QUESTIONS

Questions 6–8 are **long-answer questions** worth a total of 18 points. For long-answer questions, all of your work must be justified and your steps must be written in a clear and logical order. Clearly indicate Question numbers.

For example: **Q6(b).** [`write a fully justified solution`].

Q6. Consider the following improper integral: $\displaystyle\int_0^1 \frac{1}{x^2} e^{-2/x}\, dx$

(a) [**2 points**] Briefly explain what makes this integral improper.

(b) [**5 points**] Fully justify whether this integral converges or diverges, using methods of MAT1322 and appropriate notation. All of your steps must be justified. If it converges, find its exact value.

Q7. Let $\mathcal{R}$ be the region bounded by the curves $y = 2x$ and $y = 2\sqrt{x}$.

(a) [**4 points**] Sketch the region $\mathcal{R}$ and compute its area using an appropriate definite integral. Show all your work!

(b) [**2 points**] Let $\mathcal{S}$ be a solid whose flat base is the region $\mathcal{R}$ and whose cross-sections perpendicular to the x-axis are squares.

Find an expression for the approximate volume $V(x)$ of a thin slice of $\mathcal{S}$; this typical slice is perpendicular to the x-axis, has thickness Δx and lies between x and $x + \Delta x$. Briefly explain your answer.

(c) [**1 point**] Give the definite integral that computes the total volume V of $\mathcal{S}$. Do **not** evaluate the integral – just write it down.

Q8. A dam has a vertical face with a semi-circular door of diameter 10 m, as indicated in the figure below. The dam is filled with water to 23 m.

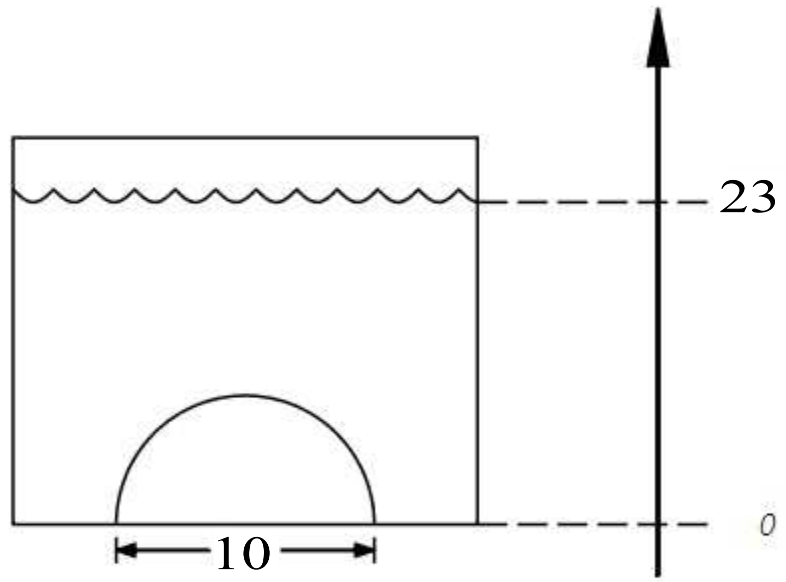

FIGURE 1

Recall: The mass density of water is 1000 kg/m^3 and acceleration due to gravity is $g = 9.8$ m/s^2.

Let y denote the height, measured from the bottom of the dam (so the bottom of the dam is at a height of $y = 0$ m).

(a) [**2 points**] Consider a thin horizontal strip of the semi-circular door, of thickness Δy m, and located about y m from the bottom of the tank.

Find an expression for the approximate area $A(y)$ of this strip, and briefly explain your approach with a diagram.

(b) [**2 points**] Again, consider a thin horizontal strip of the semi-circular door, of thickness Δy m, and located about y m from the bottom of the tank.

Find an expression for the hydrostatic force $F(y)$ acting on this strip, and briefly explain your approach.

(c) [**2 points**] With the above setup, write down the definite integral that computes the total hydrostatic force acting on the semi-circular door.

Do **not** evaluate the integral – just write it down. Briefly explain your approach.