Key Concepts and Skills

- Be able to determine the future value of an investment made today
- Be able to compute the present value of cash to be received and a future date
- Be able to compute how long it takes for and investment
- Be able to compute how long it takes for an investment to reach a desired value
- Be able to use a spreadsheet to solve time value of money problems

Basic Definitions

- *Present Value* - earlier money on a time line
    - Reflects price or value of asset or security when T=0
- *Future Value* - later money on a time line
    - Ex: investing $10,000 today how much will it be in 10 years from now?
- *Interest rate* - “”exchange rate” between money received today and mone yt be received in the future
    - Discount rate
    - Cost of capital
    - Opportunity cost of capital
    - Required return

Future Value - Example 1

- Suppose you invest $1000 for 1 year and %5 per year what is the future value in 1 year? The future value is determined by two components:
    - Interest = $1000 (0.5) = $50
    - Principal = $1000
    - Future value (in 1 year) = Principal value + interest = $1050

    - Future Value (FV) = $1000 X (1 + 0.05) = $1050

- Suppose you leave the money for another year. How much will it be 2 year from now?
    - FV = $1050 X (1 + 0.05) = $1202.50 OR 1000 (1 + 0.05)^2

Future Value: General Formula

FV = PV (1 + r)^t

- FV = Future value
- PV = Present Value
- r = Period interest rate, expressed as a decimal
- t = number of periods
- Future value interest factor = (1 + r)^t

Effects of Compounding

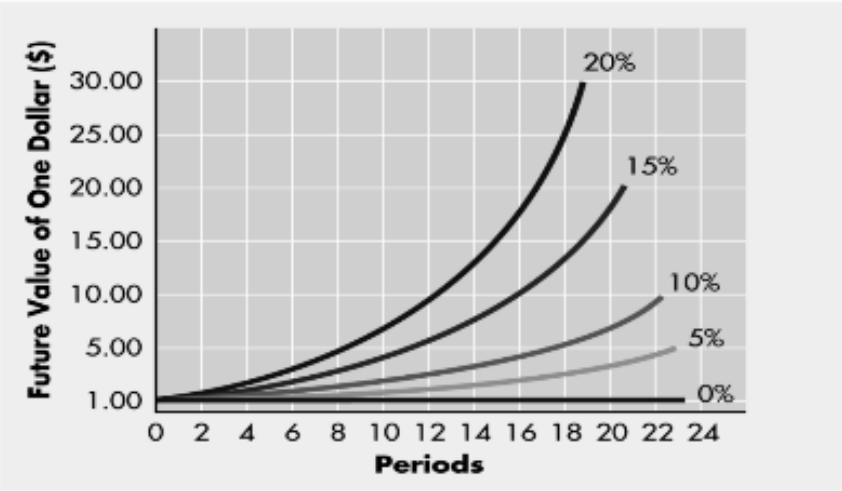
- Simple interest - earn interest on principal only
- Compound interest - Earn interest on principal and reinvested interest

Consider the previous example

- FV with *simple* interest = $1000 +$50 +$50 = $1100
  - FV = PV +(PV) (I) (t)
- FV with *compound* interest = $1000 (1.05) (1.05) = $1102.50
  - FV = PV(1+r)^t

The extra $2.50 comes from the interest of 5% earned on the $50 of interest paid in year one.

Effects of Compounding (FV of $1)



Future Value - Example 2

- Compounding over long periods of time makes a huge difference
  - Suppose today you deposited $10 at %5.5 interest
  - How much will this investment be worth in 200 years?

- FV with compound = $10 (1.055)^200 = 445,189.84
- FV with simple = $10 + (10) (0.055) (200) = $120

Comparing the impact of Simple and Compound interest on $1 invested at %10

	Simple Interest		Compound Interest	
Years	Formula	Result	Formula	Result
1	$1 (1+ 10% x 1)	$1.10	$1 (1+ 10%)^{1}	$1.1
10	$1 (1+ 10% x 10)	2	$1 (1+ 10%)^{10}	2.59
50	$1 (1+ 10% x 50)	6	$1 (1+ 10%)^{50}	117.4
100	$1 (1+ 10% x 100)	11	$1 (1+ 10%)^{100}	13,781
150	$1 (1+ 10% x 150)	16	$1 (1+ 10%)^{150}	1,617,718
200	$1 (1+ 10% x 200)	21	$1 (1+ 10%)^{200}	189,905,277
250	$1 (1+ 10% x 250)	26	$1 (1+ 10%)^{250}	22,293,142,370
1000	$1 (1+ 10% x 1000)	101	$1 (1+ 10%)^{1000}	Nobody had this fortune ever

Future Value as a General Growth Formula

- Suppose your company expects to increase unit sales of widgets by 15% per year for the next 5 years. If you currently sell 3 million widgets in one year, how many widgets do you expect to sell in 5 years.

- FV = 3,000,000 (1.15)^5 =6,034,072.56

Present Value (everything to today's dollars)

- How much do I have to invest today to have some specified amount in the future? Start with the formula of FV and rearrange
  FV = PV (1 + r)^t
- Rearrange to solve for PV:
  PV = FV / (1 + r)^t
- When we talk about the "value" of something, we are talking about the present value unless we specifically indicate that we want the future value.

Present Value - Example

- You set up a trust fund 10 years ago that is now worth $19,671.51.
- If the fund earned 7% per year, how much did you invest?

- PV = FV / (1 + r)^t
- 19,671.51 / (1.07)^10 = $1000

Present Value - Important Relationships

- For a given interest rate:
  - The longer the time period, the lower the present value
- For a given time period
  - The higher the interest rate, the smaller the present value

Discount Rate

- We often want to know the implied interest rate for an investment
- Rearrange the basic PV equation to solve for r

- FV = PV (I +r)^t
- r = (FV/PV)1/t - 1

Discount Rate - Example

- Suppose you have a 1-year old daughter and you want to provide $75,000 in 17 years towards her college education. You currently have $5,000 to invest.
- What interest rate must you earn to have the $75,000 when you need it?

- r = (FV/PV)1/t - 1

- r = (75,000/5000)^1/17 -1 = 17.27%

Finding the Number of Periods

- Start with basic equation to solve for t

- FV = PV (1 + r)^t
- FV/PV = (1 + r)^t
- ln (FV/PV) = t ln (1 + r)
- t = ln (FV/PV) / ln (1 + r)

Number of Periods - Example

- You want to buy a new apartment. You currently have $15,000 and you figure you need to have a 10% down payment. If the type of apartment you want costs about $200,000 and you can earn 7.5% per year, how long will it take before you have money for the down payment?

- t = ln (FV/PV) / ln (1 + r)
- t = ln (20,000 / 15,000) / ln (1 + 0.075) = 3.98

Quick Quiz

- How quickly will you double your money?

- An investment proposal offers you an annual compound rate of 6%. If you agree to invest $5000 today, how long will it take to double this amount?

- t = ln (10,000/5000) / ln (1 + 0.06) = 11.9 years

Spreadsheet Example

- Use the following formulas for TMV calculations
- FV (rate, nper, pmt, pv)
- PV (rate, nper, pmt, fv)
- Rate(nper, pmt, fv, pv)
- NPER(pmt, fv, pv, rate)

- … use Excel to solve the following:
    - You will need $50,000 in 10 years. If you can earn %6 interest, how much do you need to invest today?
    - You should find $27,920

Summary of FV and PV calculations

I. Symbols:

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future

r = Interest rate, rate of return, or discount rate per period—typically, but not always, one year

t = Number of periods—typically, but not always, the number of years

C = Cash amount

II. Future value of *C* invested at *r* percent for *t* periods:

$FV_t = C \times (1 + r)^t$

The term $(1 + r)^t$ is called the *future value factor.*

III. Present value of *C* to be received in *t* periods at *r* percent per period:

$PV = C/(1 + r)^t$

The term $1/(1 + r)^t$ is called the *present value factor.*

IV. The basic present value equation giving the relationship between present and future value is:

$PV = FV_t/(1 + r)^t$