MAT1341 - DGD1 solutions
January 17

1. Let $\mathbf{u} = (1,-1,0)$ and $\mathbf{v} = (2,-1,-2)$ be two vectors of $\mathbb{R}^3$.

a) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

b) Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.

c) Find a vector of $\mathbb{R}^3$ that is orthogonal to $\mathbf{v}$.

Solution :

a) $\cos(\theta) = \frac{\mathbf{u}.\mathbf{v}}{||\mathbf{u}||\,||\mathbf{v}||} = \frac{1*2+(-1)*(-1)+0*(-2)}{\sqrt{1^2+(-1)^2+0^2}\sqrt{2^2+(-1)^2+(-2)^2}} = \frac{1}{\sqrt{2}}$, so $\theta = \arccos\frac{1}{\sqrt{2}}$.

b) $\mathrm{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{v}.\mathbf{u}}{||\mathbf{v}||^2}\mathbf{v} = (2/3,-1/3,-2/3)$

c) $(x,y,z).(2,-1,-2) = 0$, so $2x-y-2z = 0$. We can choose $(1,0,1)$, for instance.

2. Find the intersection of the line $L_1 = \{(4,13,-7)+t(-1,-6,4) \mid t \in \mathbb{R}\}$ and $L_2 = \{(2,0,2)+t(0,1,-1) \mid t \in \mathbb{R}\}$.

Solution :

$$
\begin{aligned}
4-t &= 2 \quad (1)\\
13-6t &= s \quad (2)\\
-7+4t &= 2-s \quad (3)
\end{aligned}
$$

Equation (1) becomes $t = 2$. Substitute $t = 2$ in (2) to get $s = 1$. Substitute $t = 2$ and $s = 1$ in (3) to get $1 = 1$. So the intersection is $(4,13,-7)+2(-1,-6,4) = (2,1,1)$.

3. Find the distance between the point $(5,4,7)$ and the line passing through $(3,-1,2)$ and $(3,1,1)$.

Solution :
The directional vector for the line is $(3,-1,2)-(3,1,1) = (0,-2,1)$. So the equation of the line is $L = \{(3,-1,2)+t(0,-2,1) \mid t \in \mathbb{R}\}$.
The distance between any point on the line and $(5,4,7)$ is given by $||(5,4,7)-(3,-1,2)-t(0,-2,1)|| = ||(2,5+2t,5-t)|| = \sqrt{4+(5+2t)^2+(5-t)^2}$.
We therefore need to minimize the function $4+(5+2t)^2+(5-t)^2 = 54+10t+5t^2$, which has a minimum at $t = -1$.
Substitute $t = -1$ into $||(2,5+2t,5-t)||$ to get $\sqrt{49} = 7$.

4. Find a vector parametric equation for the line $2x+3y-13 = 0$ of $\mathbb{R}^2$.

Solution :
Set $x = t$, so $y = \frac{13}{3} - \frac{2}{3}t$, so $L = \{(0, 13/3) + t(1, -2/3) \mid t \in \mathbb{R}\}$.

5. Find a vector parametric equation and a Cartesian equation for the line passing through $(1,3)$ and $(2,7)$.

Solution :
For the vector parametric equation, we get $(1,3) + t((2,7) - (1,3)) = (1,3) + t(1,4)$.
For the Cartesian equation, we have $x = 1 + t$ which gives $t = x - 1$. and $y = 3 + 4t = 3 + 4(x - 1) = 4x - 1$.

6. Find all the values of k for which $(k,k,1)$ and $(k,-2,-3)$ are orthogonal.

Solution :
$(k,k,1).(k,-2,-3) = k^2 - 2k - 3 = 0$, which implies that $k = -1$ or $k = 3$.

7. Find the point of intersection of the plane $P = \{(x,y,z) \mid 2x + 2y - z = 5\}$, and the line with parametric equations $x = 4 - t, y = 13 - 6t, z = -7 + 4t$.

Solution :
Plug in $x = 4 - t$, $y = 13 - 6t$ and $z = -7 + 4t$ into the equation of P to get $2(4 - t) + 2(13 - 6t) + 7 - 4t = 5$, which turns into $t = 2$. Plug $t = 2$ into the parametric equation of the line to get $(2,1,1)$.