MATH 1210 Assignment 1 Fall 2021 **Due date: October 4**

Attempt all questions and show all your work. Some or all questions will be marked.

1. **Use mathematical induction on integer n to prove each of the following:**

   (a) $1(3)+2(3^2)+3(3^3)+\cdots+(n+1)(3^{n+1}) \;=\; \frac{1}{4}[(2n+1)(3^{n+2})+3]$ **for** $n\geq 1$ **;**

   (b) $1(4)+2(5)+3(6)+\cdots+2n(2n+3) \;=\; \frac{2}{3}(n)(2n+1)(2n+5)$ **for** $n\geq 1$ **;**

   (c) $\frac{(2n)!}{(n!)^2} > 2^n$ **for** $n\geq 2$ **;**

   (d) $a^{3n}-b^{3n}$ **is divisible by** a^2+ab+b^2**, where** a **and** b **are fixed integers and** n **is any positive integer.**

2. **Write the sum in sigma notation.**

$$3-\frac{4(5)}{\sqrt{3}}+\frac{9(7)}{\sqrt{5}}-\frac{16(9)}{\sqrt{7}}+\cdots+\frac{121(23)}{\sqrt{21}}$$

3. **Consider the sum** $(3)^2+(8)^2+(13)^2+\cdots+(15n-2)^2$**:**

   (a) Write the sum in sigma notation.

   (b) Use identities $\sum_{k=1}^{m} k = \frac{1}{2}\left[m(m+1)\right]$ **and** $\sum_{k=1}^{m} k^2 = \frac{1}{6}\left[m(m+1)(2m+1)\right]$ **to prove that**

$$(3)^2+(8)^2+(13)^2+\cdots+(15n-2)^2 = \frac{1}{2}(n)\left(450n^2+69n-35\right).$$

4. **Prove that** $\sum_{\ell=1}^{2n} \ell(\ell+1) = \frac{4}{3}\left[n(n+1)(2n+1)\right]$ **by each of the following two methods:**

   (a) **By mathematical induction on positive integer** $n\geq 1$ **.**

   (b) **By using the identities mentioned in part (b) of question** 3 **.**

5. **Evaluate** $\frac{1}{2^{90}}z^{91} + (\overline{-i})^{91} + \frac{-2+\sqrt{3}i}{i}$ **, where** $z=-\sqrt{3}+i$ **. Simplify as much as possible.**

6. **For each of the following statements, if it is true prove it, and if it is false give a counter example.**

   (a) $\frac{\overline{z}}{|z|^2} = \frac{1}{z}, \quad (z\neq 0)$ **;**

   (b) $\arg(z+\overline{z}) = 0$ **;**

   (c) $\frac{e^{4\theta^2 i}\,(e^{\theta\, i})^4}{e^{i^7}} = \cos(2\theta+1)^2 + i\sin(2\theta+1)^2$ **.**

7. **Find all fifth roots of** $z=-16\sqrt{2}-16\sqrt{2}\,i$ **. Write the roots in exponential form and use principal value of their arguments.**