Carleton UNIVERSITY

Mechanics of Solids II
EXAMINATION
December 20th 2020

DURATION: 3 **HOURS** No. of Students: 170

Department Name & Course Name: Mechanical and Aerospace Eng. MAAE 3202A & B
Course Instructor(s) Professor R. LIU

AUTHORIZED MEMORANDA
Any type of calculator and course notes only

Students **MUST** count the number of pages in this examination question paper **before** beginning to write, and report any discrepancy immediately to a proctor. This question paper has **13** pages.

This examination question paper **MAY** be taken from the examination room.
In addition to this question paper, students require: an examination booklet yes √ no □
a Scantron sheet yes □ no √

Note:

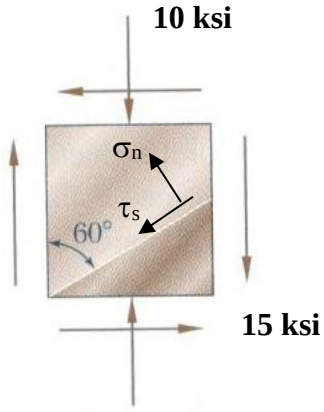
1. Answer **all** questions (Question 1 to Question 6).

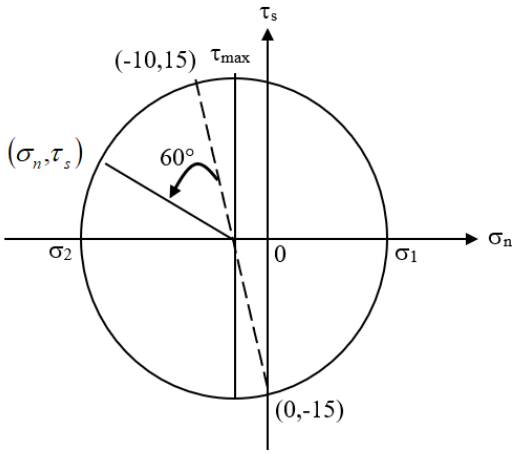
2. Write **all** answers, including those of Question 1, in the examination booklet, not on this question sheet.

Question 1 (30%)

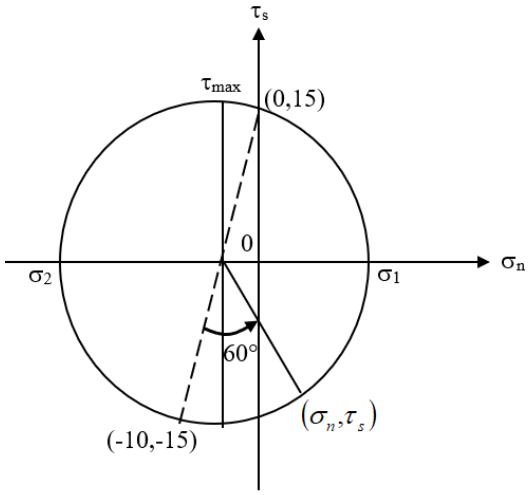
Multiple Choice Questions (Answer will be only **one** among A to D for each question.)

(1) For the 2D stress state in the figure below, which of the following stress Mohr's circles is correct?

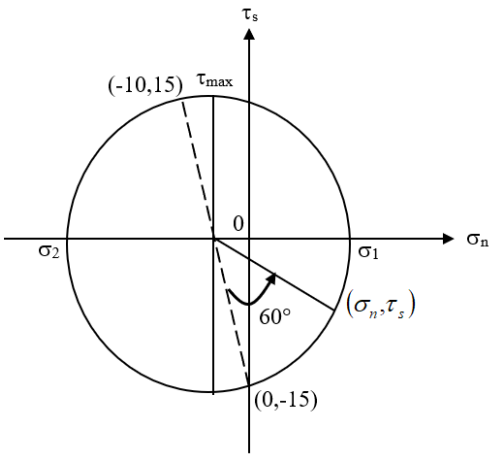




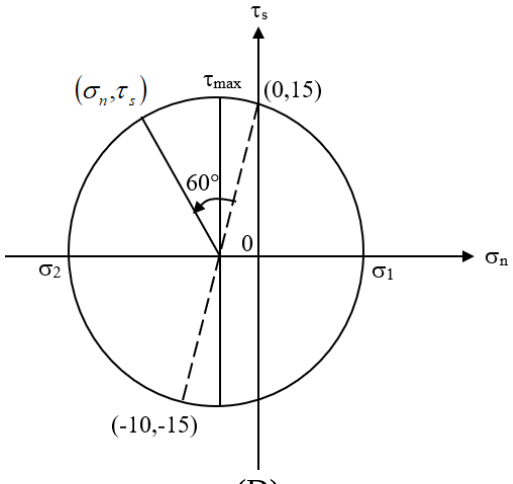
(A)



(B)

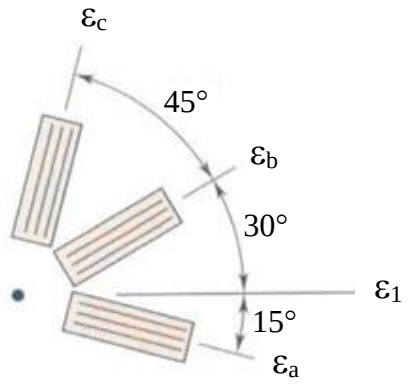


(C)



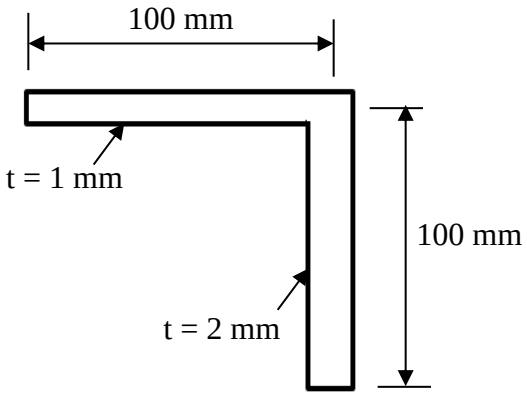
(D)

(2) To determine the strains at a point on the surface of a component, it is necessary to measure strain in three directions at the point, which can be achieved by cementing an electrical resistance rosette strain gauge to the surface, as shown in the figure below. Which of the following statements is incorrect for this measurement method?



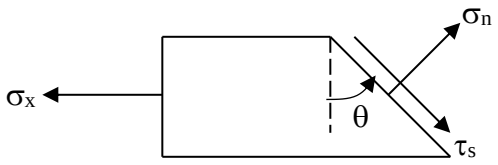
(A) The principal strains ε_1 and ε_2 can be determined from normal strain ε_a, ε_b and ε_c.
(B) The normal strain ε_a is related to the principal strains by $\varepsilon_a = \varepsilon_1 \cos^2 45° + \varepsilon_2 \sin^2 45°$.
(C) The principal stresses can be obtained from the principal strains ε_1 and ε_2 associated with the Young's modulus and Poisson's ratio of the component material.
(D) Shear strain on the principal planes is zero.

(3) A thin-walled member having an angle cross-section with variable thickness, as shown in the figure below, is subjected to a torque of 1 kN·m. What is the value of the maximum shear stress induced in the member?



(A) 4500 MPa
(B) 3330 MPa
(C) 6668 MPa
(D) 7500 MPa

(4) For the concept of stresses on an inclined plane to the direction of uniaxial loading, which of the following statements is incorrect? θ is defined to be the angle between the bar cross-section and an inclined plane, as shown in the figure below.



(A) The normal stress σ_n is always greater than the normal stress σ_x.
(B) The shear stress τ_s is always smaller than the normal stress σ_x.
(C) The shear stress τ_s has the maximum when the angle θ is 45°.
(D) The normal stresses σ_n and σ_x are equal when the angle θ is zero.

(5) For a beam subjected a moment load, as shown in the figure below, which of the following statements is incorrect?

(A) The normal stress on the beam cross-section plane is principal stress.
(B) At the neutral axis of the beam, both normal and shear stress on the beam cross-section are zero.
(C) Shear stress is zero everywhere within the beam.
(D) As a 2D stress problem, one of two principal stresses is zero at any point of top and bottom surface.

(6) Which of the following statements is incorrect for the criteria of material failure?

(A) Ductile materials fail due to yielding and brittle materials fail due to fracture.
(B) Onset of yielding is a commonly used design criterion for material failure.
(C) For a complex stress state such that a plate is subjected to stresses σ_x and σ_y, it can be suggested that the plate will fail if $\sigma_x > \sigma_Y$ or $\sigma_y > \sigma_Y$, where σ_Y is the yielding stress of the plate material.
(D) The Maximum Principal Stress Theory (Rankine) is applicable to both ductile and brittle materials, but it is usually used for brittle materials.

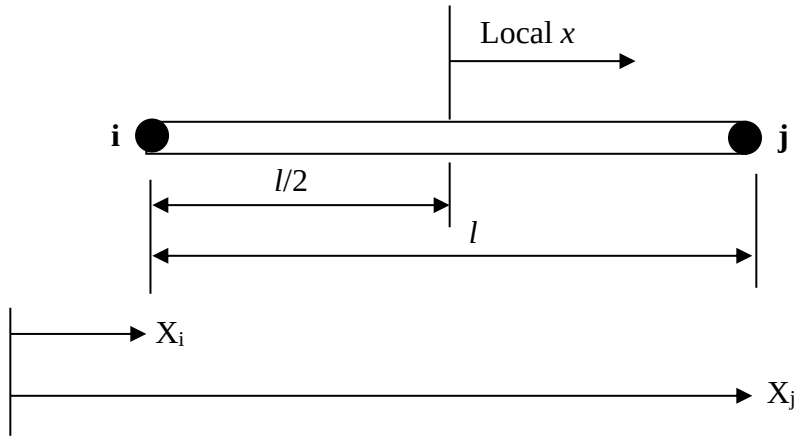
(7) For a steel thin-walled cylinder subjected to internal pressure, which of the following statements is incorrect?

(A) The first principal stress occurs on the hoop stress plane.
(B) The second principal stress occurs on the longitudinal stress plane.
(C) Shear stress is zero on the cylinder cross-section plane.
(D) Shear strain is zero everywhere within the cylinder.

(8) Which of the following statements is incorrect for 2D quadratic triangular element?

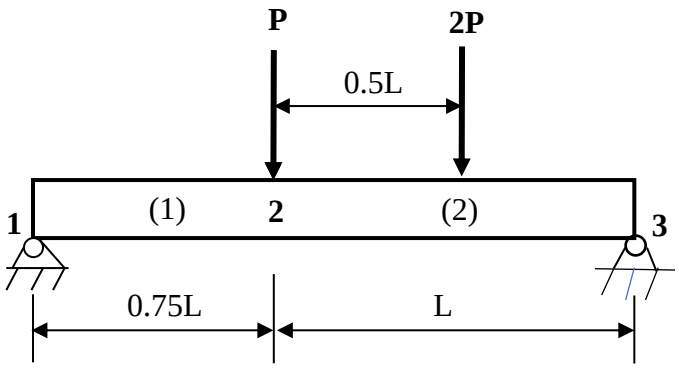
(A) The sum of the natural coordinates has a value of zero.
(B) The spatial variation of an unknown variable, for example, stress and displacement, varies nonlinearly along each edge of the element.
(C) The shape functions of corner nodes involve single natural coordinate and those of middle nodes involve two natural coordinates.
(D) The element is defined by six nodes.

(9) For a linear element, using the local coordinate system x whose origin lies at the center point of the element, as shown in the figure below, which of the following equations is correct for the shape function S_j?



(A) $S_j = \frac{1}{2} - \frac{x}{l}$
(B) $S_j = \frac{1}{4} + \frac{x}{l}$
(C) $S_j = \frac{1}{2} + \frac{x}{l}$
(D) $S_j = \frac{1}{4} - \frac{x}{l}$

(10) The beam in the figure below is subjected to two force loads P and $2P$. Which of the load matrices is correct for this beam if it is simulated using two elements?



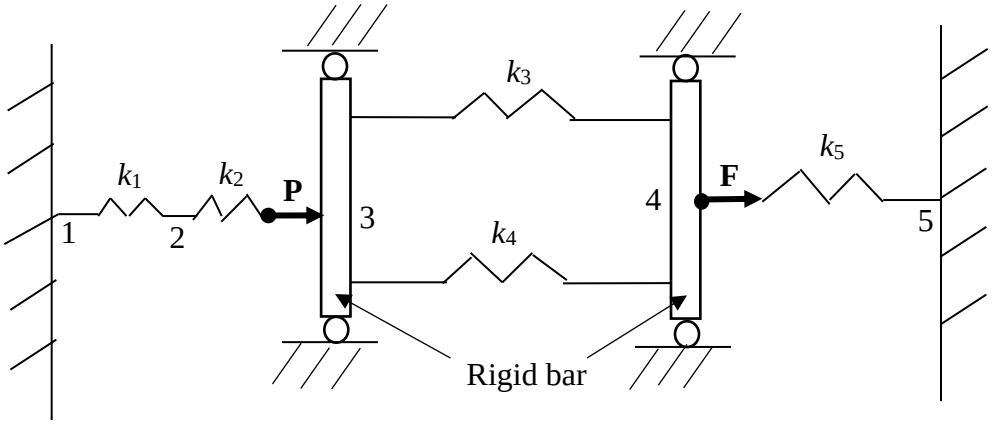
(A) $F^{(G)} = \begin{Bmatrix} -\frac{P}{2} \\ 0 \\ -P \\ -\frac{PL}{8} \\ -P \\ \frac{PL}{8} \end{Bmatrix}$

(B) $F^{(G)} = \begin{Bmatrix} 0 \\ 0 \\ -2P \\ -\frac{PL}{4} \\ -P \\ \frac{PL}{4} \end{Bmatrix}$

(C) $F^{(G)} = \begin{Bmatrix} 0 \\ 0 \\ -P \\ -\frac{PL}{4} \\ -2P \\ \frac{PL}{4} \end{Bmatrix}$

(D) $F^{(G)} = \begin{Bmatrix} -\frac{P}{2} \\ -\frac{PL}{4} \\ -2P \\ -\frac{PL}{4} \\ -P \\ \frac{PL}{4} \end{Bmatrix}$

(11) For the spring system subjected to the force loads P and F, as shown in the figure below, using FEA to determine the reactions at the constraints, which matrix equation is correct? The parameters k_1 to k_5 are spring stiffness.



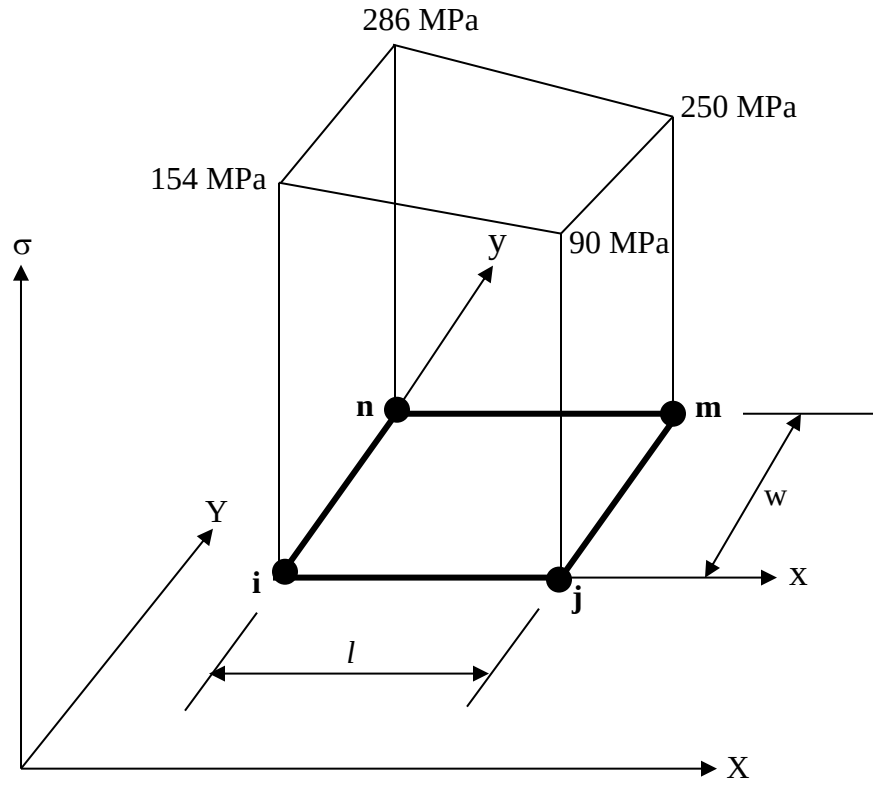
(A)
$$\begin{Bmatrix} R_1 \\ 0 \\ 0 \\ 0 \\ R_5 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2+k_3+k_4 & -k_3-k_4 & 0 \\ 0 & 0 & -k_3-k_4 & k_3+k_4+k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ P \\ F \\ 0 \end{Bmatrix}$$

(B)
$$\begin{Bmatrix} R_1 \\ 0 \\ 0 \\ 0 \\ R_5 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2+k_3 & -k_3 & 0 \\ 0 & -k_2 & k_2+k_4-k_3 & -k_4+k_3 & 0 \\ 0 & 0 & -k_4 & k_4+k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ P \\ F \\ 0 \end{Bmatrix}$$

(C)
$$\begin{Bmatrix} R_1 \\ 0 \\ 0 \\ 0 \\ R_5 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2+k_3+k_4 & -k_3-k_4+k_5 & -k_5 \\ 0 & 0 & -k_3 & k_3-k_5 & k_5 \\ 0 & 0 & -k_4 & k_4 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ P \\ F \\ 0 \end{Bmatrix}$$

(D)
$$\begin{Bmatrix} R_1 \\ 0 \\ 0 \\ 0 \\ R_5 \end{Bmatrix} = \begin{bmatrix} k_1+k_2 & -k_1-k_2 & 0 & 0 & 0 \\ -k_1-k_2 & k_1+k_2 & 0 & 0 & 0 \\ 0 & 0 & k_3+k_4 & -k_3-k_4 & 0 \\ 0 & 0 & -k_3-k_4 & k_3+k_4+k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ P \\ F \\ 0 \end{Bmatrix}$$

(12) 2D rectangular elements are used to model the stress distribution in a plate. The values of nodal stresses for an element belonging to such a plate are given in the figure below. Which of the following statements is incorrect for this element?



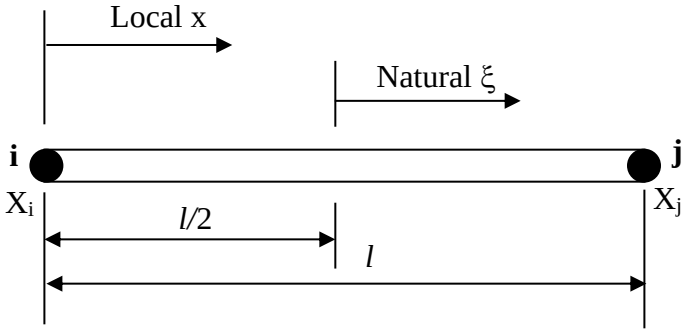
X and Y — Global coordinate system
x and y — Local coordinate system

(A) Stress decreases linearly from node *n* to node *i*.
(B) The stress at the middle of edge *ij* is greater than 90 MPa.
(C) Stress varies linearly along the diagonal *im*.
(D) Stress increases linearly from node *j* to node *m*.

(13) Which of the following statements is incorrect for 1D cubic element?

(A) The spatial variation of an unknown variable, for example, stress and displacement, can be represented by using the shape functions and the corresponding nodal values.
(B) The derivatives of the shape functions vary linearly with the position coordinate *X*.
(C) The element is defined by four nodes.
(D) The shape function has a value of unity when evaluated at its corresponding node.

(14) For a linear element, the natural coordinate system ξ originates at the center of the element, as shown in the figure below, which of the following equations is correct for representing the relationship between the local and natural coordinates?



(A) $\xi = \frac{x}{l} - \frac{1}{4}$

(B) $\xi = \frac{x}{l} + \frac{1}{2}$

(C) $\xi = \frac{2x}{l} - 1$

(D) $\xi = \frac{x}{l} - \frac{1}{2}$

(15) For FEA modelling of a beam under bending, which of the following statements is incorrect?

(A) The stiffness of a beam element is dependent on the beam cross-section geometry, element length and beam material.

(B) The constraints of a beam element are simulated with built-in ends.

(C) For a beam subjected to a uniformly distributed load, the equivalent loading is a vertical force load with the value of $wL/2$ at each node, where w is the distributed load and L is the element length.

(D) Loads are expressed at nodes in the load matrix.

Question 2 (15%)

A single horizontal force *P* of 150 lb is applied to end *D* of lever *ABD*, as shown in Figure 2. Knowing that portion *AB* of the lever has a diameter of 1.2 in.

(1) Determine the normal and shear stresses on an element located at point *H* with sides parallel to the *x* and *y* axes.
(2) Determine the principal stresses and principal planes as well as the maximum shear stress at point *H*.
(3) Draw the stress Mohr's circle.

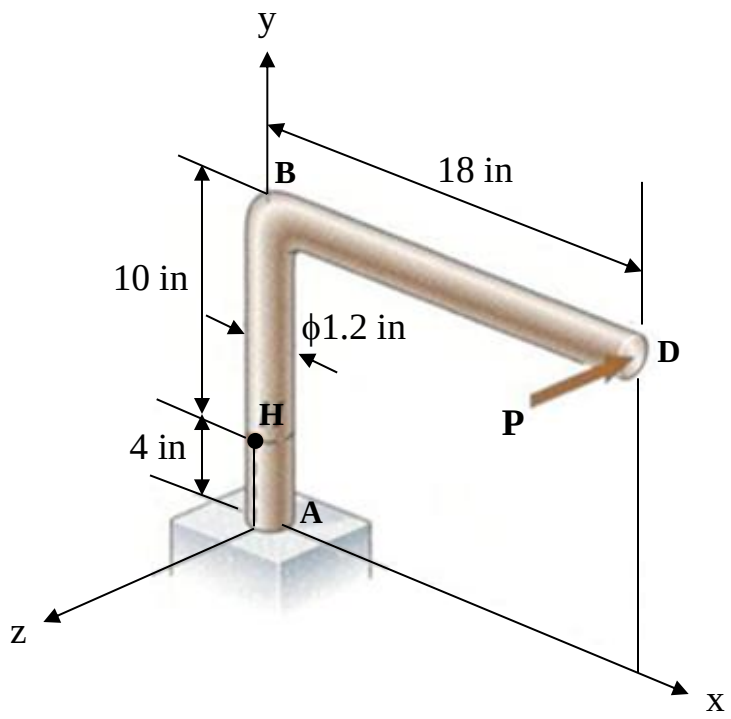
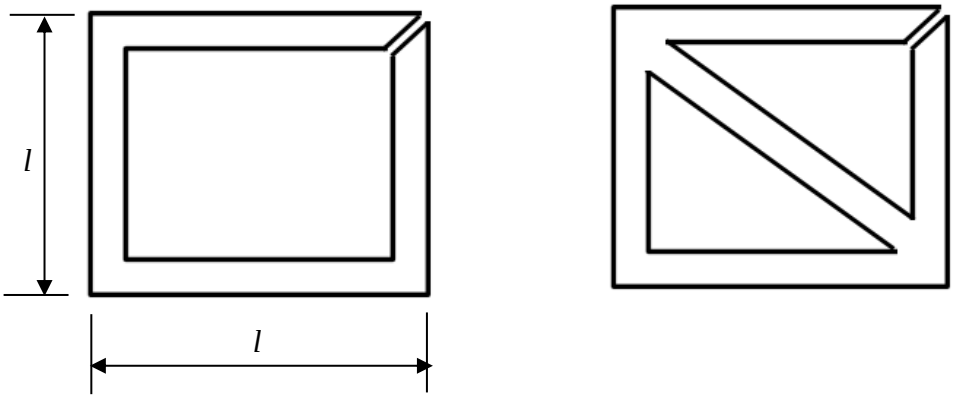


Figure 2

Question 3 (10%)

A thin-walled member having the cross-section shown in Figure 3(a) is subjected to a torque T. The member has a uniform wall thickness t. To strengthen the structure, a support is added as shown in Figure 3(b). Assuming $l = 20t$, compare the maximum shear stress and twisting angle of these two members. The dimensions of the slot are neglected.

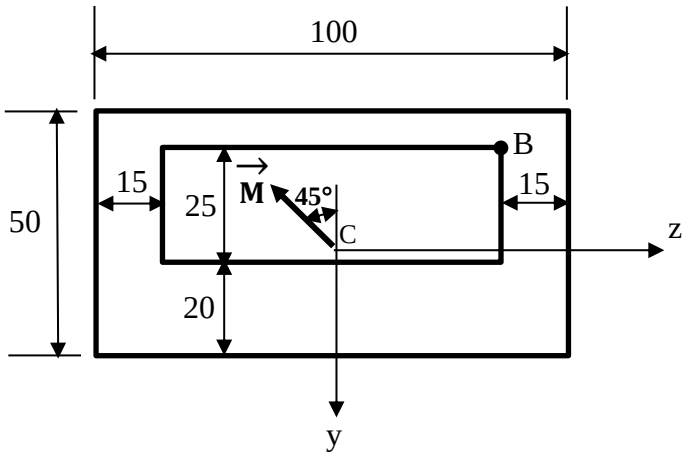


(a) (b)

Figure 3

Question 4 (20%)

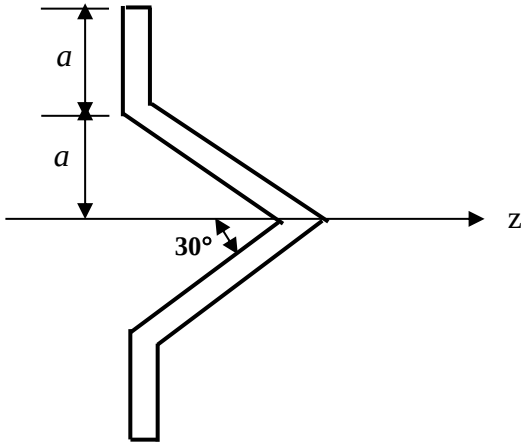
(1) For a hollow beam having a cross-section under the loading shown in Figure 4(a), if the bending moment at the cross-section M = 10 kN·m, determine the bending stress at the point B. Is it tensile or compressive?



Dimensions in mm

(a)

(2) Determine the location of the shear center for the cross-section (symmetric about the z-axis) in Figure 4(b). The load acts perpendicularly to the z-axis, the second moment of area of the section I_z is known, and the section has constant wall thickness t.



(b)

Figure 4

Question 5 (10%)

Determine the horizontal displacement of the roller at *B* and the vertical displacement at *C* of the steel truss system illustrated in Figure 5. The members are made of steel with the Young's modulus of 200 GPa. All members have the same cross-sectional area of 250 mm^2.

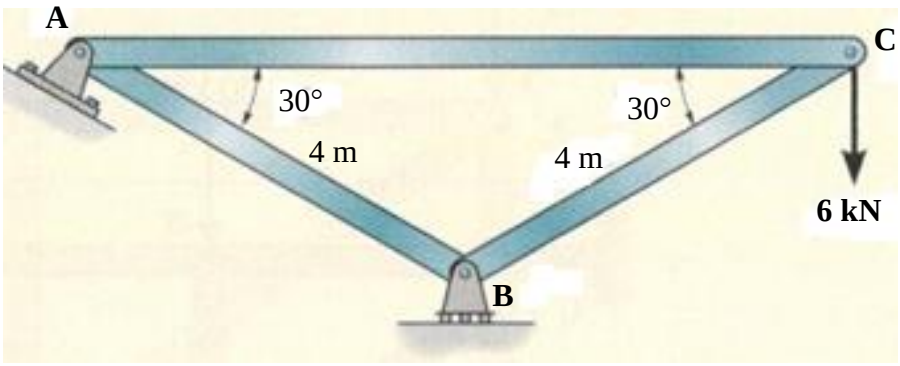


Figure 5

Question 6 (15%)

An open-ended cylindrical steel tank having an internal diameter of 4 ft is subjected to an internal pressure of 5000 psi and an external pressure of 1000 psi. The Young's modulus and Poisson's ratio of steel are 3.2×10^7 psi and 0.3, respectively.

(1) If the allowable stress of steel is 36 ksi, what is the minimum allowable outside diameter of the tank?
(2) What is the maximum tensile stress if the tank with the minimum allowable outside diameter is subjected to the internal pressure only? Determine the change in wall thickness of the cylinder in this case.