# MAT 1320 Midterm Test #3

July 23, 2015

| Question | Response |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| Total | |

Solutions
Version 1

Family Name:____________________

First Name:____________________

Student Number: ____________________

**PLEASE READ THESE INSTRUCTIONS VERY CAREFULLY**

(1) You have 80 minutes to complete the test.
(2) This is a closed book exam, and no notes of any kind are allowed. **Do not use your own scrap paper! Use the last page or the backs of pages for rough work.**
(3) The use of calculators, cell phones, pagers or any text storage or communication device is not permitted.
(4) Questions 1 – 5 are multiple choice, are worth one point each, and no part marks will be given. Please write your answers in the table above on this cover page.
(5) Questions 6,7 and 8 are long answer: you have to show clearly how you arrive at your answer. Your job is to convince me you understand why your answer is correct and what method you use.
(6) Good luck! Bonne chance!

(1) Which of the following are the critical points of $f(x) = \frac{x^2-1}{x+1}$?

A. $x = -1$

B. $x = -1, x = 1$

C. $x = 1$

D. $x = 0, x = 1$

E. $x = 0, x = -1$

F. $f$ has no critical points.

$$f'(x) = \frac{2x(x+1) - (x^2-1)}{(x+1)^2} = \frac{x^2+2x+1}{(x+1)^2} = \frac{(x+1)^2}{(x+1)^2}$$

$x = -1$ is the only point where $f'(x) = 0$ or undef$^d$, but this is not in the domain of $f$

(2) Which of the following is $\lim_{x\to 0^+} \frac{x}{\cos(x)-1}$ ?

A. 0

B. $+\infty$

C. $-\infty$

D. 1

E. $-1$

F. None of the above.

$\lim_{x\to 0^+} \frac{x}{\cos(x)-1}$ is of type $\frac{0}{0}$

L'Hospital: $\lim_{x\to 0^+} \frac{x}{\cos x - 1} = \lim_{x\to 0^+} \frac{1}{-\sin(x)} = -\infty$

(3) Consider the function $f(x) = |(x-1)^2 - 1|$. Which of the following best describes absolute and local maxima of $f$ on the closed interval $[0, 3]$ ?

A. $f$ has a local max at $x = 1$, and does not have an absolute max.

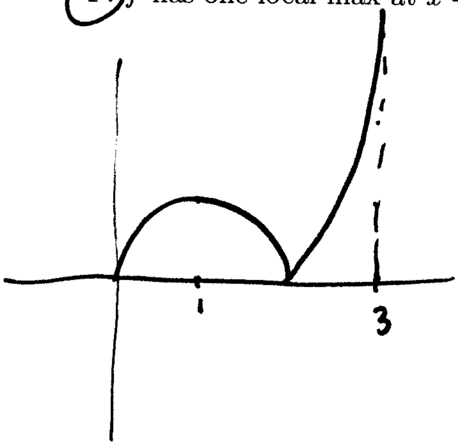
B. $f$ has a local max at $x = 3$ but not absolute max.

C. $f$ has one absolute max at $x = 1$ but has no local max.

D. $f$ has one absolute max at $x = 3$ but has no local max.

E. $f$ has one absolute max at $x = 0$ and one absolute max at $x = 3$.

F. $f$ has one local max at $x = 1$ and one absolute max at $x = 3$.




local max at $x = 1$
global max at $x = 3$

(4) What is $\lim_{x \to \pi/2^+} \tan(x) - \sec(x)$? $= \lim_{x \to \pi/2^+} \frac{\sin x}{\cos x} - \frac{1}{\cos x}$

A. 0

B. 1

C. $-1$

D. $\infty$

E. $-\infty$

F. None of the above.

$= \lim_{x \to \pi/2^+} \frac{\sin x - 1}{\cos(x)}$ type $\frac{0}{0}$

$= \lim_{x \to \pi/2^+} \frac{\cos x}{-\sin x} = \frac{0}{-1} = 0$

(5) Of the following limits, only one exists. Which one?

A. $\lim_{x\to\infty} -x\sin(x)$

B. $\lim_{x\to\infty} e^x \sin(x)$

C. $\lim_{x\to\infty} \frac{e^x}{x^2}$

D. $\lim_{x\to\infty} \frac{x}{\ln(x)}$

E. $\lim_{x\to\infty} \frac{x}{\arctan(x)-\pi/2}$

F. $\lim_{x\to\infty} \frac{1}{x}\cos(x)$. $= 0$, because

$$\underbrace{-\frac{1}{x}}_{} \leq \frac{1}{x}\cos(x) \leq \underbrace{\frac{1}{x}}_{} \quad \text{for } x>0$$

both tend to 0

(6) [**4 points**] Consider the function $f(x) = \frac{2x^3-8x}{x^2-1}$. In each of the questions below, you have to explain your method and justify your answers.

(a) Find the domain of $f$.

(b) Does $f$ have vertical asymptotes? If so, find them.

(c) Does $f$ have horizontal asymptotes? If so, find them.

(d) Does $f$ have slant asymptotes? If so, find them.

(a) $f$ is <u>not</u> defined for $x^2 = 1$, i.e. $x = \pm 1$.

Thus $\operatorname{dom}(f) = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(b) $\lim\limits_{x \to 1^+} f(x) = \frac{-6}{0} = -\infty$ So VA at $x = 1$

(also $\lim\limits_{x \to 1^-} f(x) = +\infty$)

$\lim\limits_{x \to -1^-} f(x) = +\infty$ So VA at $x = -1$

$\lim\limits_{x \to -1^+} f(x) = -\infty$

(c) No, $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} \frac{2x - 8/x}{1 - 1/x^2} = \infty$,

same for $x \to -\infty$.

(d) Yes :

$$x^2 - 1 \, / \, 3x^3 - 8x \, \backslash \, 3x$$
$$\underline{3x^3 - 3x} \; -$$
$$-5x$$

So $f(x) = 3x - \frac{5x}{x^2-1}$ and

$\lim\limits_{x \to \infty} f(x) - 3x = \lim\limits_{x \to \infty} - \frac{5x}{x^2-1} = 0$,

So the SA is $y = 3x$

(7) In the questions below, explain your method.

(a) [2 points] Find $\lim_{x\to\infty} x^{1/x}$.

Use $x = e^{\ln(x)}$ to get

$$\lim_{x \to \infty} x^{1/x} = \lim_{x \to \infty} e^{\ln(x) \cdot \frac{1}{x}}$$

Now $\lim_{x \to \infty} \ln(x) \cdot \frac{1}{x} = 0$ (L'Hopital!)

so $\lim_{x \to \infty} e^{\ln(x) \cdot \frac{1}{x}} = e^0 = 1.$

(b) [2 points] Find $\int \frac{\sin x}{1-\sin^2 x} dx$.

Use $1 - \sin^2(x) = \cos^2(x)$ to get

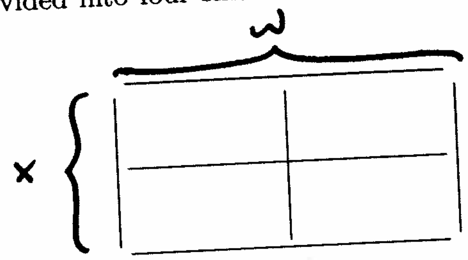
$$\int \frac{\sin x}{1-\sin^2 x} dx = \int \frac{\sin(x)}{\cos^2(x)} dx$$

Now let $u = \cos(x)$, $\frac{du}{dx} = -\sin(x)$

$$\int \frac{\sin(x)}{\cos^2(x)} dx = -\int \frac{1}{u^2} du = \frac{1}{u} + C$$

$$= \frac{1}{\cos(x)} + C \qquad (= \sec(x) + C)$$

(8) **[3 points]** A farmer wishes to fence off an area of $400m^2$. The area should be rectangular, and should be subdivided into four smaller rectangles. The following picture illustrates the shape of the fence:




Determine which width and length of the farmland result in the smallest quantity of fence needed. Explain your method.

Area $= x \cdot w = 400$

fence length: $\ell = 3x + 3w$

(we wish to minimize $\ell$)

From $xw = 400$ get $w = \dfrac{400}{x}$, so

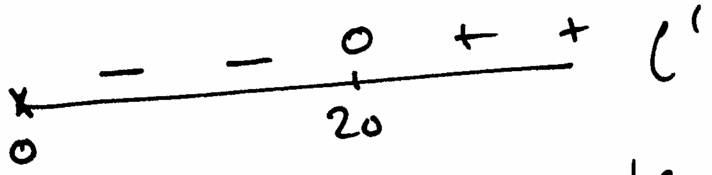
$$\ell = 3x + \frac{1200}{x}.$$

$$\ell' = 3 - \frac{1200}{x^2}.$$ Set equal to 0

$$\ell' = 0 \iff \frac{1200}{x^2} = 3$$

$$\iff x^2 = 400$$

$$\iff x = \pm 20$$ (ignore neg. sol$^n$.)




Thus $\ell$ has an abs. min. for $x = 20$.

$\Rightarrow$ optimal size is $x = 20$, $w = 20$.