Lecture 9 Note - *Centre of Gravity and Centroids*

Textbook Chapter 9.1 - [Lecture Notes]

Before we start: What are integrals?

- The derivative of a function gives us its slope. The integral of a function gives us its area. If *f(x)* is a graph of velocity vs time, *d/dx f(x)* is acceleration, and ∫ *f(x)dx* is displacement.
- If we find the integral (area) of a function, we can find its centroid (centre). Why do we need this? Well, imagine you're trying to find the centre of mass of an object. It would be in the centre, obviously, but how would you find that if you have a weird curve shape? That's where the integral comes in. It cuts up the curve into tiny slices that are so small we can pretend each is rectangular. It's easy to find the area/centroid of a rectangle, so if you split a curve into rectangles and add them up, you can find its entire area/midpoint.
- The derivative of x^3 is $3x^2$. Likewise, the integral of x^3 is $x^4/4$ +C, using $\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$
- The signs ∫ and ***dx*** are like a pair of brackets: They go together and enclose something. Once you calculate the integral of the stuff inside, these "brackets" can be removed.
- How do these equations work, exactly? Leave that to the math professors. For Statics, all we need to know is the power rule for integrals, highlighted above. ↑

What *is* the centroid, and the Centre of Gravity?

- The centroid and centre of gravity are basically the same thing. The centroid is the geographic centre of an area, AKA the midpoint. So the centroid of some object would also be its centre of gravity, mass, and volume (or area if it's 2D).
- The centroid is located at ($\bar{x}$, $\bar{y}$, $\bar{z}$), a point in an object where $\bar{x}$ is the distance of the centroid from the origin along the *x*-axis, $\bar{y}$ the distance along the *y*-axis, and so on…
- The distance of a particle inside the body from the origin is located at ($\tilde{x}$, $\tilde{y}$, $\tilde{z}$). What particle? Any particle we want to measure — if we break up the shape into smaller pieces for easier calculations, each piece will have a centroid at ($\tilde{x}$, $\tilde{y}$, $\tilde{z}$).

Centre of Mass Formula: (Same as centre of gravity but with *m* for mass instead of *w* for weight):

- $\bar{x} = \frac{\int \tilde{x} dW}{\int dW} = \frac{g \int \tilde{x} dm}{g \int dm} = \frac{\int \tilde{x} dm}{\int dm}$ (*W = mg,* ∴ d*W* = d*mg. g cancels out.*) This formula is for the distance of the centre of mass along the *x* axis, but you can swap *m* for area, volume, etc. to find the centroid of anything, and *x* can be switched with *x/y/z.*

How to calculate the centroid of an object?

- **2D Object (area):** Same as CoM formula but with A for area instead of M for mass. **3D Object (volume)**: Same as CoM formula but with V for volume. You get the idea…