[1] **1.** Find the domain of the function $g(x) = \dfrac{xe^{\sqrt{1-2x}}}{x^2-9}$.

solution: We need to make sure that the expression in the square root is non-negative, and we need to make sure that the denominator is not zero. So we must have $1-2x \geq 0$ which gives $x \leq 1/2$ and we also need to have $x^2-9 \neq 0$ which gives $x \neq \pm 3$. So the domain is $(-\infty,-3) \cup (-3,1/2]$.

[2] **2.** Using the definition of the derivative, find $\dfrac{\mathrm{d}}{\mathrm{d}x}\sqrt{x+7}$.

solution:

$$\lim_{h\to 0}\frac{\sqrt{(x+h)+7}-\sqrt{x+7}}{h}$$
$$=\lim_{h\to 0}\frac{\sqrt{(x+h)+7}-\sqrt{x+7}}{h}\times\frac{\sqrt{(x+h)+7}+\sqrt{x+7}}{\sqrt{(x+h)+7}+\sqrt{x+7}}$$
$$=\lim_{h\to 0}\frac{\left(\sqrt{(x+h)+7}\right)^2-\left(\sqrt{x+7}\right)^2}{h\left(\sqrt{(x+h)+7}+\sqrt{x+7}\right)}$$
$$=\lim_{h\to 0}\frac{((x+h)+7)-(x+7)}{h\left(\sqrt{(x+h)+7}+\sqrt{x+7}\right)}$$
$$=\lim_{h\to 0}\frac{h}{h\left(\sqrt{(x+h)+7}+\sqrt{x+7}\right)}$$
$$=\lim_{h\to 0}\frac{1}{\left(\sqrt{(x+h)+7}+\sqrt{x+7}\right)}$$
$$=\frac{1}{\left(\sqrt{x+0+7}+\sqrt{x+7}\right)}$$
$$=\frac{1}{(2\sqrt{x+7})}$$

[4] **3.** Evaluate each of the limits. Show your work!

a) $\displaystyle\lim_{x\to 2}\frac{x-2}{2x^2-4x}$

solution:

$$\begin{aligned}\lim_{x\to 2}\frac{x-2}{2x^2-4x} &= \lim_{x\to 2}\frac{x-2}{2x(x-2)}\\ &= \lim_{x\to 2}\frac{1}{2x}\\ &= \frac{1}{2(2)}\\ &= \frac{1}{4}\end{aligned}$$

b) $\lim_{x\to\infty}\frac{x^3-x-7}{2x^3+\sqrt{x}}$

solution:

$$\begin{aligned}\lim_{x\to\infty}\frac{x^3-x-7}{2x^3+\sqrt{x}} &= \lim_{x\to\infty}\frac{x^3-x-7}{2x^3+\sqrt{x}}\times\frac{1/x^3}{1/x^3}\\ &= \lim_{x\to\infty}\frac{1-1/x^2-7/x^3}{2+1/x^{5/2}}\\ &= \frac{1-0-0}{2+0}\\ &= \frac{1}{2}\end{aligned}$$

[2] **4.** Suppose that $x^3+xy-e^y=5$. Find $y'=\frac{dy}{dx}$ in terms of x and y.

solution:

$$\begin{aligned}x^3+xy-e^y &= 5\\ 3x^2+y+xy'-e^yy' &= 0\\ xy'-e^yy' &= -3x^2-y\\ y' &= \frac{-3x^2-y}{x-e^y}\end{aligned}$$

[2] **5.** Let $f(x)=-2x^3+x$. Find the equation of the line tangent to f at the point $x=1$.

solution: First we find the derivative: $f'(x)=-6x^2+1$. The slope is $f'(1)=-5$. The value of the function is $f(1)=-1$. Then we have the line.

$$\begin{aligned}(y-y_0) &= m(x-x_0)\\ (y-(-1)) &= (-5)(x-1)\\ y &= -5x+4\end{aligned}$$

[6] **6.** Differentiate each of the following. Show your work! You do not need to simplify your answer.

a) $u(t) = (t^2 + \pi)^7 (e^t + 1)$

solution:

$$u'(t) = 7(t^2 + \pi)^6 (2t)(e^t + 1) + (t^2 + \pi)^7 (e^t)$$

b) $w(r) = \ln\left(\sqrt{1 - r^2}\right)$.

solution:

$$w'(r) = \frac{1}{\sqrt{1 - r^2}} \frac{1}{2} (1 - r^2)^{-1/2} (-2r)$$

c) $g(x) = \dfrac{\tan(x)}{x^2 + 2^x}$.

solution:

$$g'(x) = \frac{\sec^2(x)(x^2 + 2^x) - \tan(x)(2x + 2^x \ln(2))}{(x^2 + 2^x)^2}$$

[2] **7.** Use logarithmic differentiation to find the derivative of $f(x) = \left(x^2 + 2\right)^{\sin(x)}$. Give your answer in terms of x (not f), but you do not need to simplify.

solution: First take ln of both sides.

$$\begin{aligned} \ln(f) &= \ln\left(\left(x^2 + 2\right)^{\sin(x)}\right) \\ &= (\sin(x)) \ln(x^2 + 2) \end{aligned}$$

Now differentiate implicitly, using product rule and chain rule. Remember to substitute f at the end.

$$\frac{1}{f} f' = (\cos(x)) \ln(x^2 + 2) + (\sin(x)) \frac{1}{x^2 + 2} (2x)$$

$$f' = \left(x^2 + 2\right)^{\sin(x)} (\cos(x)) \ln(x^2 + 2) + (\sin(x)) \frac{1}{x^2 + 2} (2x)$$

[3] **8.** A rectangle is changing width and height continuously. When the height is 20cm and the area is 100cm^2, the width is increasing at a rate of 1cm/s and the area is increasing at a rate of $10\text{cm}^2/\text{s}$. At this moment, what is the rate of change of the height?

solution: Consider a rectangle of width w, height h and area A. We know that $A = wh$. Differentiating we get $\dfrac{dA}{dt} = \dfrac{dw}{dt} h + w \dfrac{dh}{dt}$. When the height is 20 and the area is 100 the width must be $100/20 = 5$. At this moment we know $\frac{\mathrm{d}w}{\mathrm{d}t} = 1$ and $\frac{\mathrm{d}A}{\mathrm{d}t} = 10$.

$$\begin{aligned}\frac{\mathrm{d}A}{\mathrm{d}t} &= \frac{\mathrm{d}w}{\mathrm{d}t}h + w\frac{\mathrm{d}h}{\mathrm{d}t} \\ 10 &= (1)(20) + (5)\frac{\mathrm{d}h}{\mathrm{d}t} \\ \frac{\mathrm{d}h}{\mathrm{d}t} &= \frac{10-20}{5} \\ &= -2\end{aligned}$$

Note that the height is decreasing, since its derivative is negative.