

midterm test

Academic matters:

- There are five questions. Each is worth five marks.
- You are allowed to use material from the course britespace, as well as your own notes you have made.
- You are not allowed to use any other materials, including but not limited to other books, webpages, online resources, or help from any other person.
- You may use the result of one question (even one you were not able to solve) as part of your solution to another question.
- You may not use without proof the result of an assignment question.

Technological matters:

- The test will be run over Zoom.
You should have your camera on during the test so you can be seen; this is intended to be like normal proctoring during an exam. No recordings will be made.
- You may ask me questions during the test.
Indicate in the chat if you have a question (message to me only, not to everyone). I will move you to a breakout room where we can talk without disturbing the other students. Afterwards I will move you back to the regular Zoom meeting..
- You should leave your microphone off, unless we are in the “questions” breakout room.
- You should leave your sound on in case I have to make an announcement.
- At the end you should take pictures of your work with a camera, and submit that as a .pdf file to britespace.
- This whole process is still a work in progress. I think this will all work, but if something goes wrong, I will fix it somehow.

1. a) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2y \\ y - x \end{bmatrix}$. Is T linear?
b) Let $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 - y \\ x \end{bmatrix}$. Is S linear?

2. Suppose that $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly independent in some vector space. Show that $\{\mathbf{x} - \mathbf{y}, \mathbf{y} - \mathbf{z}, \mathbf{z}\}$ is linearly independent.

3. Define $T \in \mathcal{L}(\mathcal{P}_2, \mathbb{R}^3)$ with $T(p) = \begin{bmatrix} p(0) \\ p'(0) \\ p''(0) \end{bmatrix}$, where p' and p'' are the first and second derivatives of p .

Is T an isomorphism?

(You can use the fact that T is linear without proving it.)

4. Suppose that V is a vector space and that M is a subspace of V . Let \mathcal{U} be the set of all $T \in \mathcal{L}(V)$ such that $T(\mathbf{x}) \in M$ for all $\mathbf{x} \in M$.
a) Show that \mathcal{U} is a subspace of $\mathcal{L}(V)$.
b) Show that if $T, S \in \mathcal{U}$ then $T \circ S \in \mathcal{U}$.

(You can use the fact that $\mathcal{L}(V)$ is a vector space without proving it.)

5. Suppose that $T \in \mathcal{L}(\mathbb{R}^5)$, with $\dim(\ker(T)) = 2$.
a) Find $\dim(\text{im}(T))$.
b) Show that there is some vector $\mathbf{x} \in \mathbb{R}^5$ such that $T(T(\mathbf{x})) \neq \mathbf{0}$.