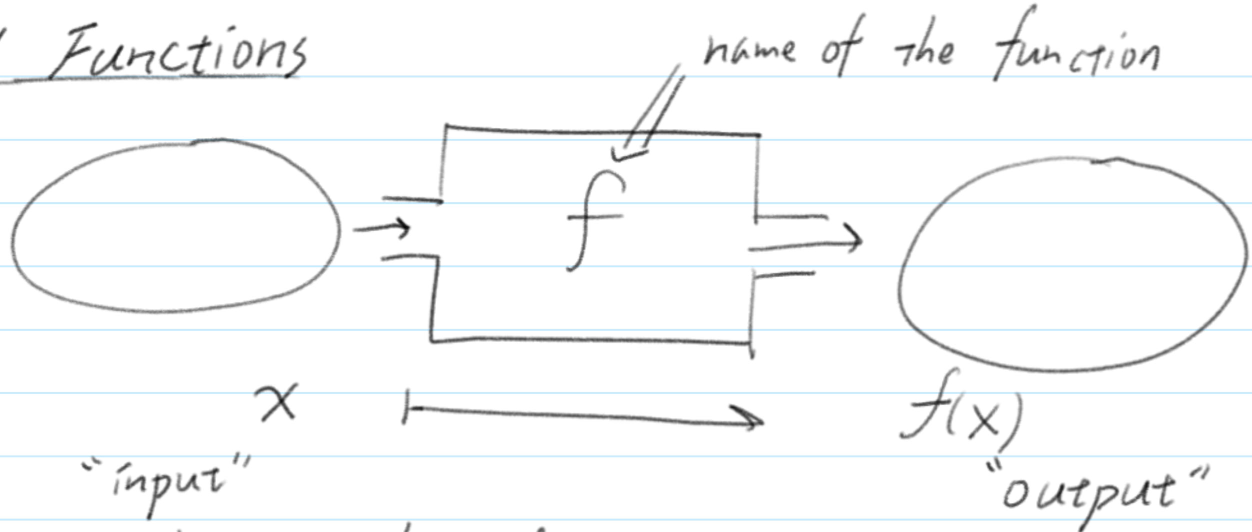
Lecture 01 (1.1-1.3)

2018年8月15日 下午 01:28

1.1 Functions



We mostly consider functions whose inputs and outputs are real numbers.

Example $f(x) = x^2+2$.

a) Evaluate $f(-1)$ b) Solve $f(x) = 6$

Sol.

a) $f(-1) = (-1)^2+2 = 1+2 = \boxed{3}$

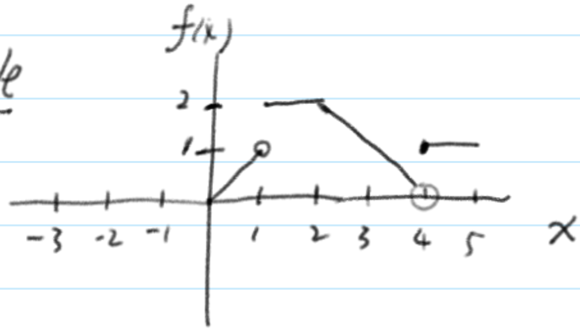
b) $f(x) = 6 \iff x^2+2 = 6$

$\iff x^2 = 4$

$\iff \boxed{x = 2 \text{ or } -2}$ (or "$x = \pm 2$")

Graphs as functions

Example



a) Evaluate $f(4)$

b) Solve $f(x) = 2$

Sol. a) $f(4) = 1$

b) Which values of x give the output 2?

Answer: $1 \le x \le 2$

(or interval notation: $[1, 2]$)

Domain of a function

Domain: The set of possible input values

Range: " output values

Two common restrictions:

① can't divide by zero;

② can't take square root (or nth root for n even) of a negative number.

Example Find the domain of each function:

a) $f(x) = 2\sqrt{x-1}$ b) $g(x) = \frac{3}{2-2x}$

Sol.

a) Need to have $x-1 \ge 0$, so $\boxed{x \ge 1}$

(interval notation: $[1, +\infty)$)

b) Cannot have $2-2x = 0$. This happens precisely when $x = 1$. Have to exclude 1

So $\boxed{x<1 \text{ or } x>1}$ or $\boxed{(-\infty, 1) \cup (1, +\infty)}$

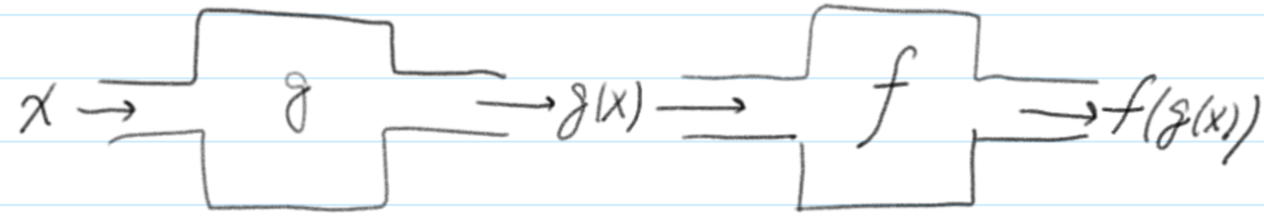
✡ Read about "interval notations" (p14)

✡ $\sqrt{4} = 2$. NOT $\sqrt{4} = \pm 2$!!

✡ Read the "vertical line test" (p10)

1.2 Operations on Functions

Composition of functions



The composition is again a function. We write it as $f \circ g$. In other words, $f \circ g(x) = f(g(x))$.

Example $f(t) = t^2 + t$ and $g(x) = 2x - 3$.
Evaluate $f \circ g(2)$ and $g \circ f(2)$.

Sol.

- $g(2) = 2 \cdot 2 - 3 = 1$
  $f \circ g(2) = f(g(2)) = f(1) = 1^2 + 1 = 2.$
- $f(2) = 2^2 + 2 = 6$
  $g \circ f(2) = g(f(2)) = g(6) = 2 \cdot 6 - 3 = 9.$

☆ $f \circ g$ and $g \circ f$ are in general very different!

We can also write $f \circ g$ as a formula:

$$\begin{aligned} f \circ g(x) = f(g(x)) &= f(2x-3) \\ &= (2x-3)^2 + (2x-3) \\ &= (4x^2 - 12x + 9) + (2x-3) \\ &= 4x^2 - 10x + 6. \end{aligned}$$

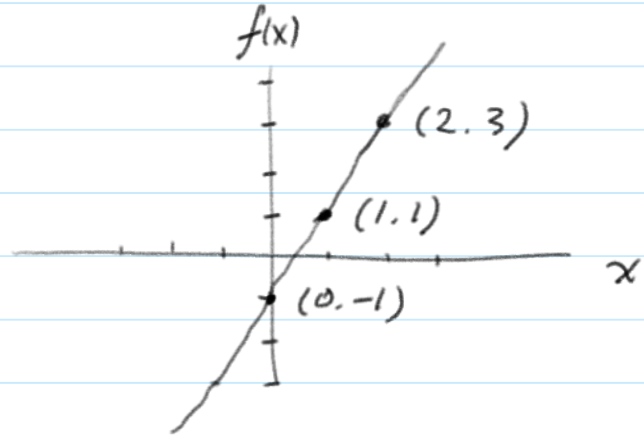
(Try to evaluate $f \circ g(2)$ using this formula)

✲ Read about "Toolkit Functions (p.13)

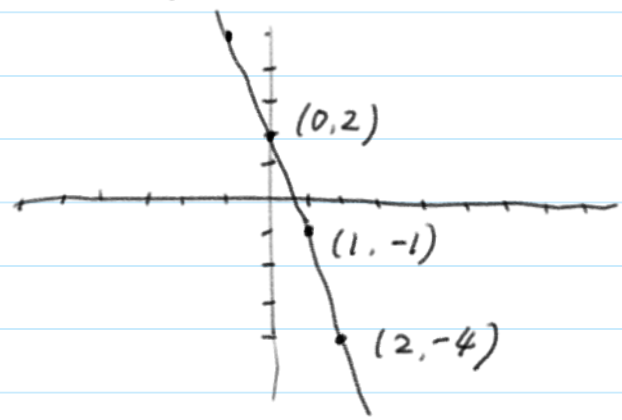
✲ Read about "Transformations of Functions" and how to sketch their graphs (p.22-27)

1.3 Linear Functions

<u>Example</u> $f(x) = 2x - 1$



$g(x) = -3x + 2$



A <u>linear function</u> is a function f of the form

$$f(x) = mx + b$$

- The graph of a linear function is a line (hence the name "linear")
- b is precisely the value $f(0)$, the so-called "y-intercept".
- m is the <u>slope</u>. When we move 1 unit to the right, the graph move m unit(s) vertically.

- m is the <u>slope</u>. When we move 1 unit to the right, the graph move m unit(s) vertically.
- $f(x) = mx+b$ is called the "slope-intercept form"

<u>Example</u> Write a formula for the linear function passing through $(1, 2)$ and $(-2, 4)$.

<u>Sol.</u>

The slope m is given by

$$m = \frac{\text{change in output}}{\text{change in input}} = \frac{4-2}{-2-1} = \frac{2}{-3} = -\frac{2}{3}$$

So $f(x) = -\frac{2}{3}x + b$. Now f passes through $(1, 2)$, so

$$\begin{aligned} & f(1) = 2 \\ \Leftrightarrow \quad & -\frac{2}{3} + b = 2 \\ \Leftrightarrow \quad & b = 2 + \frac{2}{3} \\ \Leftrightarrow \quad & b = \frac{6}{3} + \frac{2}{3} = \frac{6+2}{3} = \frac{8}{3} \end{aligned}$$

We conclude

$$f(x) = -\frac{2}{3}x + \frac{8}{3}$$

- Check that indeed $f(-2) = 4$.

<u>Example</u> Think about cell phone cases.

price $\uparrow$ $\Rightarrow$ less people buying (lower demand)

price $\uparrow$ $\Rightarrow$ manufacturers produce more (higher supply)

People model demand and supply by linear functions.

$$\text{demand} = d(p) = -0.7p + 8.7 \qquad (p = \text{price})$$
$$\text{supply} = s(p) = 1.2p + 0.5$$

The equilibrium price is the price when $d(p) = s(p)$.
In our case,

$$d(p) = s(p)$$
$$-0.7p + 8.7 = 1.2p + 0.5$$
$$8.2 = 1.9p$$
$$p = \frac{8.2}{1.9} \approx 4.32.$$