University of Ottawa
MAT1300-E. Fall 2018, Midterm Exam 1
Monday, October 15^{th}, 2018

First Name **Version A** ____________________

Family Name **Solution** ____________________

Do **not** write your student ID number on this front page. Please write your student ID number in the space provided on the second page.

Take your time to read the entire paper before you begin to write, and read each question carefully. Make note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 80 minutes to complete this exam.
- This is a closed book exam, and no notes of any kind are allowed.
- No calculators are allowed.
- This exam consists of 8 questions: 5 are multiple choice and 3 are long answer.
  - For the 5 multiple choice questions, only the chosen answer entered in the grid on the second page will be marked.
  - For the 3 long answer questions, the correct answer requires justification written legibly and logically: you must convince me that you know your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

  By signing below, you acknowledge that you have ensured that you are complying with the above statement.

  Signature :____________________

- Good Luck!

Student Number: ______________________, Total marks: _______________ out of 25

Grid to enter your multiple choice answers:

Question	1	2	3	4	5
Answer	C	A	B	B	B

Grid for the marker:

	Multiple Choice (10 marks)	Q6 (5 marks)	Q7 (5 marks)	Q8 (5 marks)	Total (25 marks)
Marks					

QUESTION 1. (2 points) Solve for x.

$$\ln(5x) - \ln(6 - x^2) = 0$$

Among the following answers, enter the correct one's letter into the grid on page 2.

A) $x = -6$ only

B) $x = -6$ and $x = 1$

C) $x = 1$ only

D) $x = 6$ and $x = -1$

E) $x = 6$ only

$$\begin{aligned}
\ln\left(\frac{5x}{6-x^2}\right) &= 0 \\
\frac{5x}{6-x^2} &= 1 \\
5x &= 6 - x^2 \\
x^2 + 5x - 6 &= 0 \\
(x-1)(x+6) &= 0 \\
x &= 1 \text{ or } -6
\end{aligned}$$

Since $x = -6$ is not in the domain, the answer is C.

QUESTION 2. (2 points) For what value of a is the following function continuous at $x = 2$?

$$f(x) = \begin{cases} x^2 + ax - 11 & \text{if } x > 2 \\ \frac{3}{x+1} & \text{if } x \leq 2 \end{cases}$$

Among the following answers, enter the correct one's letter into the grid on page 2.

A) $a = 4$ B) $a = \dfrac{8}{3}$ C) $a = -6$

D) $a = 2$ E) $a = -\dfrac{1}{4}$

$$\begin{aligned} \lim_{x \to 2^-} f(x) &= \frac{3}{2+1} = 1 \\ \lim_{x \to 2^+} f(x) &= 2^2 + 2a - 11 = 2a - 7 \\ f(2) &= \frac{3}{2+1} = 1. \end{aligned}$$

We need $2a - 7 = 1$, or $a = 4$. The answer is A.

QUESTION 3. (2 points) Find the equation of the tangent line of the function $f(x) = x\sqrt{2x+5}$ at $x = 2$.

Among the following answers, enter the correct one's letter into the grid on page 2.

A) $y = \frac{13}{3}x - \frac{8}{3}$ B) $y = \frac{11}{3}x - \frac{4}{3}$ C) $y = \frac{17}{3}x - \frac{2}{3}$

D) $y = \frac{13}{3}x + \frac{7}{3}$ E) $y = \frac{11}{3}x + \frac{2}{3}$

$$\begin{aligned} f'(x) &= \sqrt{2x+5} + x \cdot \frac{d}{dx}\left((2x+5)^{1/2}\right) \\ &= \sqrt{2x+5} + x \cdot \frac{1}{2}(2x+5)^{-\frac{1}{2}} \cdot 2 \\ &= \sqrt{2x+5} + \frac{x}{\sqrt{2x+5}}. \\ f'(2) &= 3 + \frac{2}{3} = \frac{11}{3}. \end{aligned}$$

The tangent line has the form $y = \frac{11}{3}x + b$ and passes throught the point $(2, f(2)) = (2, 6)$. Therefore

$$6 = \frac{11}{3} \cdot 2 + b \Longrightarrow b = -\frac{4}{3}.$$

The answer is B.

QUESTION 4. (2 points) Consider the function

$$f(x) = x - e^x.$$

Which of the following statements is **true** (there is only one)? Enter the correct one's letter into the grid on page 2.

A) f has a global maximum at $x = 1$.

B) f has a global maximum at $x = 0$.

C) f has a global minimum at $x = 1$.

D) f has a global minimum at $x = 0$.

E) f has no global maximum on the real line.

Since $f'(x) = 1 - e^x$, the only critical number for f is $x = 0$. We compute $f''(x) = -e^x$ and $f''(0) = -1$. By the second derivative test, f has a local maximum at $x = 0$. Since it is the only critical number, it has to be a global maximum.

As $f'(x) < 0$ when x is large, the function f decreases without bound. We conclude that f has no global minimum

The answer is B.

QUESTION 5. (2 points) Calculate

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 4}.$$

Among the following answers, enter the correct one's letter into the grid on page 2.

A) 0

B) $\frac{1}{4}$

C) $\frac{1}{2}$

D) 1

E) The limit does not exist.

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 4} = \lim_{x \to 2} \frac{(x-1)(x-2)}{(x+2)(x-2)} = \lim_{x \to 2} \frac{x-1}{x+2} = \frac{1}{4}.$$

The answer is B.

QUESTION 6. (5 points) Using only the limit definition of the derivative, calculate $f'(x)$ where

$$f(x) = \frac{1}{4x}.$$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{4(x+h)} - \frac{1}{4x}}{h} && \text{(2 points up to here)} \\
&= \lim_{h\to 0} \frac{1}{h} \cdot \frac{[4x - 4(x+h)]}{4(x+h)(4x)} \\
&= \lim_{h\to 0} \frac{1}{h} \cdot \frac{-4h}{4(x+h)(4x)} \\
&= \lim_{h\to 0} \frac{-1}{(x+h)(4x)} && \text{(2 more points up to here)} \\
&= \frac{-1}{4x^2}.
\end{aligned}$$

QUESTION 7. (5 points) Consider the function

$$f(x) = x^3 - 3x^2 - 9x + 7.$$

a) (2 points) Find all critical numbers of f.

b) (1 point) Determine the open interval(s) on which f is increasing.

c) (2 points) For each critical number, determine whether f has a local maximum, local minimum, or neither.

a)

$$\begin{aligned} f'(x) &= 3x^2 - 6x - 9 \\ &= 3(x^2 - 2x - 3) \\ &= 3(x-3)(x+1) \end{aligned}$$

The critical numbers are $x = 3$ and $x = -1$.

b)

interval	$(-\infty, -1)$	$(-1, 3)$	$(3, +\infty)$
sign of $f'(x)$	$+$	$-$	$+$

The function f is increasing on $(-\infty, -1) \cup (3, +\infty)$.

c) By the first derivative test,

- f has a local maximum at $x = -1$, and
- f has a local minimum at $x = 3$.

To the marker: if the student's computation in (a) was incorrect, but the analysis in (b) and/or (c) are correct, give full credits for (b) and/or (c).

QUESTION 8. (5 points) Consider the function

$$g(x) = \ln(x^2 + 4).$$

a) (4 points) Determine the open interval(s) on which g is concave down.

b) (1 point) Determine the open interval(s) on which g' is increasing.

a)

$$\begin{aligned} g'(x) &= \frac{1}{x^2+4} \cdot (2x) \\ &= \frac{2x}{x^2+4}. \\ g''(x) &= \frac{2 \cdot (x^2+4) - (2x) \cdot (2x)}{(x^2+4)^2} \\ &= \frac{2x^2 + 8 - 4x^2}{(x^2+4)^2} \\ &= \frac{-2x^2+8}{(x^2+4)^2} \\ &= \frac{-2(x^2-4)}{(x^2+4)^2} \\ &= \frac{-2(x+2)(x-2)}{(x^2+4)^2}. \qquad \text{(2 points up to here)} \end{aligned}$$

interval	$(-\infty, -2)$	$(-2, 2)$	$(2, +\infty)$
sign of $g''(x)$	$-$	$+$	$-$

We conclude that g is concave down on the interval $(-\infty, -2) \cup (2, +\infty)$.

b) The function g' is increasing when its derivative, namely g'', is positive. Therefore g' is increasing on the interval $(-2, 2)$.

To the marker: if the computation of g'' was incorrect, but the analysis of concavity and/or part (b) is correct, give 2 points to part (a) and/or full credit (1 point) for part (b).

Space for additional work