University of Ottawa
MAT 2379
Introduction to Biostatistics
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Solution to class problems (Week 5)
Chapter 4 (part 2): Section 4.2 and Chapter 5 (part 1): Section 5.1

Example 1. (left-handed) Let X be the number of left-handed persons in a group of 6. X has a binomial distribution with $n = 6$ trials and probability of success $p = 0.13$. We have to calculate

$$P(X = 1) + P(X = 2) = \begin{pmatrix} 6 \\ 1 \end{pmatrix} (0.13)^1 (0.87)^5 + \begin{pmatrix} 6 \\ 2 \end{pmatrix} (0.13)^2 (0.87)^4$$

$$= 0.3888 + 0.1542 = 0.5340$$

The answer is B.

Example 2. (West Nile Virus) a) Let X be the number of units which test positives in a pool of 6. X is a binomial random variable with $n = 6$ and $p = 1/500 = 0.002$. The required probability is

$$P(X \geq 1) = 1 - P(X = 0) = 1 - (1 - 0.002)^6 = 0.01194$$

b) There are 216/6=36 pools. By part a), each one has probability 0.01194 of testing positive. Let Y be the number of pools (among the 36) which will test positive. Y is a binomial random variable with $n = 36$ and $p = 0.01194$. The required probability is

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - (1 - 0.01194)^{36} = 0.351$$

The answer is B.

Example 3. (binomial R output) We have:

$$P(37 < X < 43) = P(38 \leq X \leq 42) = P(X \leq 42) - P(X \leq 37) = 0.4108588 - 0.1197826 = 0.291076$$

The answer is D. The wrong answer A is obtained computing

$$P(X \leq 43) - P(X \leq 37) = 0.4850622 - 0.1197826 = 0.3652796$$

Example 4. (variance) The variance of X is

$$\begin{aligned} \mathsf{Var}(X) &= E[X^2] - (E[X])^2 = \int_{-1}^{1} 1.5\, x^4\, dx - \left(\int_{-1}^{1} 1.5\, x^3\, dx \right)^2 \\ &= 1.5\, \frac{x^5}{5}\bigg|_{-1}^{1} - \left(1.5\, \frac{x^4}{4}\bigg|_{-1}^{1} \right)^2 = 0.6 - (0)^2 = 0.6 \end{aligned}$$

The answer is C.

Example 5. (liver cancer) Since X is measured in years, we need to transform the data in years: 6 months=1/2 years and 3 months=1/4 years. The desired probability is

$$P\left(\frac{1}{4} \leq X \leq \frac{1}{2}\right) = \int_{1/4}^{1/2} 2(1-x)dx = (2x - x^2)|_{1/4}^{1/2} =$$

$$2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 = \frac{5}{16}.$$

The answer is D.

Example 6. (Pareto) For any $x > 1$, we have:

$$F(x) = \int_1^x 2/u^3\, du = 2\,\frac{u^{-2}}{-2}\bigg|_1^x = 1 - 1/x^2.$$

We need to solve the equation $F(x_0) = 1/2$. This equation becomes $1 - \frac{1}{x_0^2} = 1/2$. We get $x_0^2 = 1/2$ and hence $x_0 = \sqrt{2}$. The answer is B.