University of Ottawa
MAT 2379
Introduction to Biostatistics
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Solution to class problems (Week 4)
Chapter 3 (part 2): Sections 3.4-3.5 and Chapter 4 (part 1): Section 4.1

Example 1. (mice) Let A be the event that the randomly chosen mouse shows signs of nervousness, and E be the event that the mouse has an induced tumor. We know that $P(E) = 0.47$ and $P(E') = 0.53$. Also, we know that $P(A|E) = 0.65$ and $P(A|E') = 0.30$. Using the law of total probability, we get:

$$P(A) = P(A|E)P(E) + P(A|E')P(E') =$$

$$(0.65)(0.47) + (0.30)(0.53) = 0.3055 + 0.1590 = 0.4645.$$

By the Bayes' rule, the probability that the mouse has an induced tumor given that it shows signs of nervousness is:

$$P(E|A) = \frac{P(A|E)P(E)}{P(A)} = \frac{0.3055}{0.4645} = 0.6577$$

The answer is D.

Example 2. (colorblind) Let A be the event "the person is a man" and B the event "the person is colorblind". We know that $P(A) = 0.49$, $P(B) = 0.06$ and $P(A \cap B) = 0.04$. We have

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.04}{0.49} = 0.08.$$

Since $P(B|A) = 0.08 \neq 0.06 = P(B)$, we conclude that colorblindness is not independent of gender. The answer is C.

Example 3. (medical) (a) Let A be the event that the student prefers a multiple choice exam, and B be the event that the student intends to pursue medical studies. From the table, we infer that:

$$P(A \cap B) = \frac{66}{119}, \quad P(A \cap B') = \frac{10}{119}$$

$$P(A' \cap B) = \frac{28}{119}, \quad P(A' \cap B') = \frac{15}{119}$$

Hence

$$P(B) = P(A \cap B) + P(A' \cap B) = \frac{66}{119} + \frac{28}{119} = \frac{94}{119}$$

The desired conditional probability is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{66/119}{94/119} = \frac{66}{94} = 0.70$$

(b) We have:

$$P(A) = P(A \cap B) + P(A \cap B') = \frac{66}{119} + \frac{10}{119} = \frac{76}{119} = 0.64$$

Since $P(A) \neq P(A|B)$, A and B are not independent. Alternatively, one may check that $P(A \cap B) \neq P(A)P(B)$. The answer is B.

Example 4. (yellow)

$$E(X) = 0(1/27) + 1(6/27) + 2(12/27) + 3(8/27) = 2.$$

The answer is C.

Example 5. (robin) We first have to calculate the mean of X:

$$\mu_X = 3(0.25) + 4(0.55) + 5(0.2) = 3.95$$

The variance of X is given by:

$$\sigma_X^2 = 3^2(0.25) + 4^2(0.55) + 5^2(0.2) - (3.95)^2 = 16.05 - 15.6025 = 0.4475$$

The standard deviation of X is:

$$\sigma_X = \sqrt{0.4475} = 0.669$$

The answer is C.

Example 6. (discrete) First, we find the table of the probability mass function of X:

x	0	1	2	3	4
$P(X = x)$	0.5	0.1	0.1	0.2	0.1

Using this table, we compute the mean and variance of X:

$$E(X) = 0(0.5) + 1(0.1) + 2(0.1) + 3(0.2) + 4(0.1) = 1.3$$

$$\text{Var}(X) = 0^2(0.5) + 1^2(0.1) + 2^2(0.1) + 3^2(0.2) + 4^2(0.1) - (1.3)^2 = 3.9 - 1.69 = 2.21$$

The answer is E.