MAT1320 Calculus I Midterm 1 SOLUTIONS October 6, 2021 11:30 AM

MULTIPLE-CHOICE QUESTIONS

Your answers to multiple-choice questions do not need to be justified. You may write your scrap work on your paper but it will not be graded. When you reach your answer, clearly indicate the question number and write the letter of your response beside the question number.

For example: (*write out your scrap work, but it will not be graded*)

(*clearly indicate your final choice*) **Q1.** [letter of your choice]

Q1. [2 points] Find the inverse of $f(x) = \dfrac{8x+5}{4x+2}$.

Solution: **B**

A. $f^{-1}(x) = \dfrac{4x-4}{3x-8}$ **B.** $f^{-1}(x) = \dfrac{5-2x}{4x-8}$ **C.** $f^{-1}(x) = \dfrac{4-8x}{3x-4}$

D. $f^{-1}(x) = \dfrac{8x-5}{2-4x}$ **E.** $f^{-1}(x) = \dfrac{6-3x}{2x-8}$ **F.** $f^{-1}(x) = \dfrac{8x-6}{2x-3}$

Q2. [2 points] The domain of $f(x) = \dfrac{1}{4-\ln(3x)}$ is the set of all real numbers x such that...

Solution: **C**

A. $x > 0$ and $x \neq \frac{e^5}{3}$ **B.** $x > 0$ and $x \neq \frac{e^3}{5}$ **C.** $x > 0$ and $x \neq \frac{e^4}{3}$

D. $x > 0$ and $x \neq \frac{e^3}{4}$ **E.** $x > 0$ and $x \neq \frac{e^5}{4}$ **F.** $x > 0$ and $x \neq \frac{e^4}{5}$

Q3. [2 points] Find the value of the constant k for which the following function is continuous:

$$f(x) = \begin{cases} \dfrac{x^2-4x-21}{x+3} & \text{if } x < -3 \\ k + x & \text{if } x \geq -3 \end{cases}$$

Solution: **F**

A. $k = -10$ **B.** $k = -5$ **C.** $k = -1$ **D.** $k = -3$ **E.** $k = -6$ **F.** $k = -7$

G. There is no such value of k.

Q4. [2 points] Consider the function $f(x) = (2x+1)e^x$.

Find the point x where the tangent line to the graph of f is <u>horizontal</u>.

Solution: **A**

A. $x = -\frac{3}{2}$ **B.** $x = -\frac{3}{4}$ **C.** $x = -\frac{7}{3}$ **D.** $x = -\frac{7}{4}$ **E.** $x = -\frac{2}{4}$ **F.** $x = -\frac{2}{7}$

G. There is no such value of x.

Q5. [2 points] Suppose $f(t)$ and $g(t)$ are differentiable functions such that

$$f(-2) = 2 \qquad g(-2) = 6$$
$$f'(-2) = 5 \qquad g'(-2) = 3$$

Find $H'(-2)$ for the function $H(t) = f(t)g(t) - t^3$.

Solution: **E**

A. 3 **B.** 15 **C.** 36 **D.** 7 **E.** 24 **F.** 20

LONG-ANSWER QUESTIONS

For long-answer questions, all of your work must be justified and your steps must be written in a clear and logical order. Clearly indicate Question numbers.

For example: **Q6(a).** [write a fully justified solution].

Q6. [6 points] Evaluate each of the following limits. For each, you must show all your steps, and use appropriate limit laws and algebraic methods seen in class. If a limit does not exist, you must justify this conclusion. Note: the limit in b) is a one-sided limit.

a) $\displaystyle\lim_{x\to 4}\left(\frac{1}{x-4} - \frac{8}{x^2-16}\right)$ b) $\displaystyle\lim_{x\to 6^-}\frac{x^2-9x+18}{|6-x|}$ c) $\displaystyle\lim_{t\to -\infty}\frac{6t-7}{\sqrt{9t^4+4}}$

Solution:

a)

$$\begin{aligned}
\lim_{x\to 4}\left(\frac{1}{x-4}-\frac{8}{x^2-16}\right) &= \lim_{x\to 4}\left(\frac{1}{x-4}-\frac{8}{(x-4)(x+4)}\right)\\
&= \lim_{x\to 4}\left(\frac{1(x+4)}{(x-4)(x+4)}-\frac{8}{(x-4)(x+4)}\right)\\
&= \lim_{x\to 4}\left(\frac{1(x+4)-8}{(x-4)(x+4)}\right)\\
&= \lim_{x\to 4}\left(\frac{x-4}{(x-4)(x+4)}\right)\\
&= \lim_{x\to 4}\left(\frac{1}{(x+4)}\right)\\
&= \frac{1}{4+4}=\frac{1}{8}
\end{aligned}$$

Solution:

b) Since $x \to 6^-$ (from the left), we have $x < 6$. Thus, $6-x > 0$ so $|6-x| = 6-x$. Using this simplification of the absolute value term, we get:

$$\begin{aligned}
\lim_{x\to 6^-}\frac{x^2-9x+18}{|6-x|} &= \lim_{x\to 6^-}\frac{x^2-9x+18}{6-x} && (\text{since } 6-x>0 \Rightarrow |6-x| = 6-x)\\
&= \lim_{x\to 6^-}\frac{(x-6)(x-3)}{-(x-6)}\\
&= \lim_{x\to 6^-} -(x-3)\\
&= -(6-3) = -3.
\end{aligned}$$

Solution:

c)

$$\begin{aligned}
\lim_{t\to -\infty}\frac{6t-7}{\sqrt{9t^4+4}} &= \lim_{t\to -\infty}\frac{6t-7}{\sqrt{t^4(9+\frac{4}{t^4})}}\\
&= \lim_{t\to -\infty}\frac{6t-7}{\sqrt{t^4}\sqrt{9+\frac{4}{t^4}}}\\
&= \lim_{t\to -\infty}\frac{6t-7}{|t^2|\sqrt{9+\frac{4}{t^4}}}\\
&= \lim_{t\to -\infty}\frac{t(6-\frac{7}{t})}{t^2\sqrt{9+\frac{4}{t^4}}}\\
&= \lim_{t\to -\infty}\frac{(6-\frac{7}{t})}{t\sqrt{9+\frac{4}{t^4}}}\\
&= \lim_{t\to -\infty}\frac{(6-0)}{t\sqrt{9+0}}\\
&= 0
\end{aligned}$$

Q7. [3 points] Use the limit <u>definition of the derivative</u> (first principles) to find $f'(x)$ for the function

$$f(x) = \sqrt{2x^2+7}$$

Show all your steps, and use appropriate limit laws and algebraic methods seen in class.

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{2(x+h)^2+7}-\sqrt{2x^2+7}}{h} \\
&= \lim_{h\to 0} \left(\frac{\sqrt{2(x+h)^2+7}-\sqrt{2x^2+7}}{h}\right)\left(\frac{\sqrt{2(x+h)^2+7}+\sqrt{2x^2+7}}{\sqrt{2(x+h)^2+7}+\sqrt{2x^2+7}}\right) \\
&= \lim_{h\to 0} \frac{2(x+h)^2+7-(2x^2+7)}{h(\sqrt{2(x+h)^2+7}+\sqrt{2x^2+7})} \\
&= \lim_{h\to 0} \frac{2(x^2+2xh+h^2)+7-(2x^2+7)}{h(\sqrt{2(x+h)^2+7}+\sqrt{2x^2+7})} \\
&= \lim_{h\to 0} \frac{2h(2x+h)}{h(\sqrt{2(x+h)^2+7}+\sqrt{2x^2+7})} \\
&= \lim_{h\to 0} \frac{2(2x+h)}{\sqrt{2(x+h)^2+7}+\sqrt{2x^2+7}} \\
&= \frac{2(2x+0)}{\sqrt{2(x+0)^2+7}+\sqrt{2x^2+7}} \\
&= \frac{2(2x)}{2\sqrt{2x^2+7}} \\
&= \frac{2x}{\sqrt{2x^2+7}}
\end{aligned}$$

Q8. [6 points] Use the rules of differentiation to find the first derivative of each of the following functions. You do not need to simplify your answer.

a) $y = (23x + 8\cos^2(x))^{-3}$ b) $g(\theta) = e^{\tan\theta + \sqrt{2}}$ c) $f(x) = \dfrac{(5x+e^x)}{(4x+5)^2}$

Solution:

a) $y' = (-3)(23x + 8\cos^2(x))^{-4}(23 + 8(2)\cos(x)(-\sin(x)))$

Solution:

b) $g'(\theta) = (e^{\tan\theta+\sqrt{2}})(\sec^2\theta + 0)$

Solution:

c) $f'(x) = \frac{(5+e^x)(4x+5)^2 - (5x+e^x)(2)(4x+5)(4+0)}{(4x+5)^4}$