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# ST[redacted] Lab Notes

**<u>Text Reference:</u> 2.1, 2.2**

**Sample Spaces and Events (2.1)**

<u>Definitions</u>:

**Experiment** - An action or process where the result is uncertain.

**Sample Space** - The set of all outcomes of an experiment.

**Event** - Any collection of outcomes from the sample space. (A subset of the sample space)

**Simple Event** - Consists of a single outcome. (A single element of the sample space)
**Compound Event** - Consists of multiple outcomes. (A non-empty set of elements from the sample space)

<u>Set Notation and Definitions (review)</u>:

1. $\emptyset \rightarrow$ The null/empty set.
2. $A \subset B \rightarrow$ The set $A$ is a subset of the set $B$. All elements in the set $A$ are also part of the set $B$
3. $A \cup B \rightarrow$ Union of the sets $A$ and $B$. Denotes all the elements that are a part of set $A$ **OR** set $B$.
4. $A \cap B \rightarrow$ Intersection of the sets $A$ and $B$. Denotes all elements that are a part of both $A$ **AND** $B$.
5. $A' \rightarrow$ The complement of $A$. All elements that are in the Universal space $S$ that are **NOT** in $A$.
6. Two sets are **disjoint** or **mutually exclusive** if $A \cap B = \emptyset$
7. De Morgan's laws: For any sets $A$ and $B$,
   (a) $(A \cup B)' = A' \cap B'$
   (b) $(A \cap B)' = A' \cup B'$

**Axioms, Interpretations and Properties of Probability (2.2)**

<u>Definitions</u>:

**Frequency** - The number of times a certain outcome occurs in a finite number of repetitions of a random experiment.

**Relative Frequency** - The ratio of the frequency of an outcome divided by the number of times the experiment is repeated.

**Probability** - If $n$ [finite] equally likely possibilities [outcomes], of which one must occur and $s$ are considered "successes" [i.e., satisfy the conditions of a specified event, $E$], then the probability of a success is given by

$$P(E) = \frac{s}{n}$$

Example: repeated rolls of a fair die will show that a 1 will appear 1 out of 6 times - so the probability is $\frac{1}{6}$

<u>Axioms of Probability</u>:

Suppose $S$ is a sample space.

1. $0 \leq P(A) \leq 1$ for any event $A \subset S$
2. $P(S) = 1$
3. If $A_1,\ A_2, \ldots,\ A_k$ are mutually exclusive events in $S$, then

$$P(A_1 \cup A_2 \cup \ldots \cup A_k) = \sum_{i=1}^{k} P(A_i) = P(A_1) + P(A_2) + \ldots + P(A_k)$$

1. **Properties of Probability:**

   (a) If $A$ and $B$ are mutually exclusive, then $P(A \cap B) = 0$

   (b) If $A$ and $B$ are exhaustive, then $P(A \cup B) = 1$

   (c) If $A \subset B$, then $P(A) \leq P(B)$

   (d) General Additive Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

   (e) Probability of a Complement: $P(A') = 1 - P(A)$

   (f) De Morgan's laws: $P((A \cup B)') = P(A' \cap B')$ and $P((A \cap B)') = P(A' \cup B')$

   (g) Law of Total Probability: $P(A) = P(A \cap B) + P(A \cap B')$

   **Example**: Suppose that $P(A) = 0.5, P(B) = 0.3$, and $P(A \cap B) = 0.2$.

   i) Find $P(A \cup B)$

   $$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.3 - 0.2 = 0.6$$

   ii) Find $P(A' \cap B')$

   $$P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.6 = 0.4$$

2. **Conditional Probability**

   Consider the events $A$ and $B$ where $P(B) > 0$. Then the probability of $A$ given that the condition(s) of event $B$ are satisfied [i.e. the conditional probability of $A$ given $B$] is:

   $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

   Note:

   The above gives us a general multiplicative rule as well: $P(A \cap B) = P(B) \cdot P(A|B)$, if $P(B) \neq 0$

   $$P(A \cap B) = P(A) \cdot P(B|A), \text{ if } P(A) \neq 0$$

   **Example:** Suppose in a class of 50 students that 15 are math majors (MA), 10 are computing majors (CP), and 3 of those are double majors in both.

   (a) Find the probability that a randomly selected student is a math major given that the student selected is at least a computing major.

   $$P(MA|CP) = \frac{P(MA \cap CP)}{P(CP)} = \frac{3/50}{10/50} = \frac{3}{10}$$

   (b) Are the events "Math Student" and "Computing Student" independent?

   $$P(MA \cap CP) = \frac{3}{50} \quad \text{and} \quad P(MA)P(CP) = \frac{15}{50}\left(\frac{10}{50}\right) = \frac{3}{50}$$

   Events are independent since $P(MA \cap CP) = P(MA)P(CP)$

3. **Independent Events**

Two events $A$ and $B$ are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$ or $P(A|B) = P(A)$ or $P(B|A) = P(B)$

Notes:

(a) If $A$ and $B$ are independent events, then the following pairs of events are also independent:

i. $A$ and $B'$

ii. $A'$ and $B$

iii. $A'$ and $B'$

(b) Events $A$, $B$, and $C$ are mutually independent if and only if the following conditions hold:

i. $A$, $B$, and $C$ are pairwise independent and

ii. $P(A \cap B \cap C) = P(A)P(B)P(C)$

4. **Law of Total Probability**

If events $B_1, B_2, ..., B_m$ are mutually exclusive and exhaustive events, then for another event $A$:

$$P(A) = \sum_{i=1}^{k} P(B_i) \cdot P(A|B_i)$$

Note that, as $P(B_i) \cdot P(A|B_i) = P(B_i \cap A)$ this statement is equivalent to $P(A) = \sum_{i=1}^{k} P(B_i \cap A)$

5. **Bayes' Theorem**

If events $B_1$, $B_2$, ..., $B_m$ are mutually exclusive of which one must occur, then for $k = 1, 2, ..., m$:

$$P(B_k|A) = \frac{P(B_k) \cdot P(A|B_k)}{\sum_{i=1}^{m} [P(B_i) \cdot P(A|B_i)]} = \frac{P(B_k) \cdot P(A|B_k)}{P(A)} \text{ (from the Law of Total Probability)}$$

**Example:** Bob takes either the bus or subway to work each day (each equally likely to occur). If he takes the bus or subway, there is a 60% and 20% chance he will be late, respectively. Determine the probability that Bob took the bus to work on a day that he is late.

Let $B, S, L$ represent the events bus, subway, and late, respectively.

$$P(B|L) = \frac{P(B)P(L|B)}{P(B)P(L|B) + P(S)P(L|S)} = \frac{0.5\,(0.6)}{(0.5)\,(0.6) + (0.5)\,(0.2)} = 0.75$$

**R and RStudio:**

R is open source statistical software/language that is widely used in industry to work with data while R Studio is software that is intended to make coding in R more user-friendly. Please note that in order to use R Studio, you must have both R and RStudio installed. Please see the "R/RStudio" folder under "Content" on MyLS for more information on downloading the two pieces of software.