Name: ____________________ Grading #:____ Student ID#: _______________

Carleton University
Faculty of Engineering
ECOR 2050

Midterm #1 – Version 1

February, 2020

Instructions: This is a closed book exam.
You may use scientific (non-programmable) calculators.
You may use the provided formula sheet and table.
Define all events/random variables, state all assumptions
For credit, you must show all of your work and write units and conclusion statements.

You have 60 minutes to complete the exam.

☺ *Good luck!*

Question	Value	Score
1	10	
2	12	
3	8	

TOTAL:	**/30**

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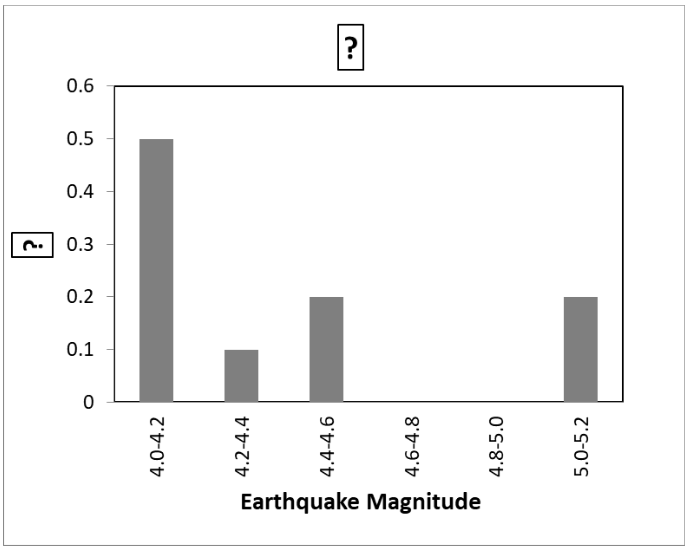
1. In the past 20 years (from 2000), there have been 10 earthquakes within a 250km radius of Ottawa that had magnitudes ≥ 4. The data are summarized in the table below.

Date	2000/08/06	2002/04/20	2003/10/12	2006/02/25	2010/06/23	2011/09/18	2012/10/10	2012/11/06	2013/05/17	2013/05/17
Magnitude	4.0	5.2	4.2	4.5	5.2	4.1	4.5	4.2	4.4	4.1

Earthquake magnitude (≥4.0) within a 250 km radius of Ottawa, since 2000.

a) The following graph is plotted to demonstrate some properties of the data set. Provide a descriptive title and appropriate y-axis label for the plot. (3 marks)

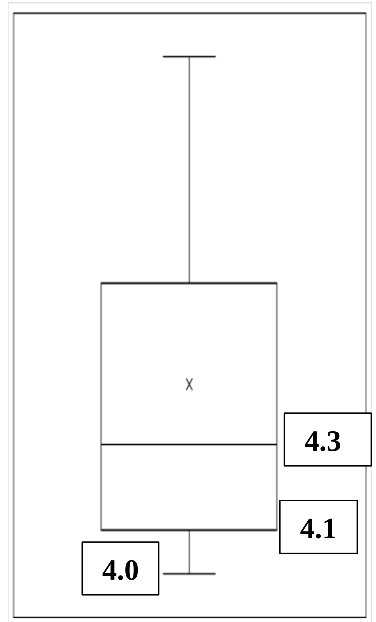


Title:
Frequency Histogram of Earthquake magnitude (≥4.0) within a 250 km radius of Ottawa, since 2000

Axis:
Relative Frequency or Probability

Name: ____________________ Grading #:____ Student ID#: _______________

b) The following Box and Whisker plot is plotted to demonstrate some properties of the dataset. Complete the plot by adding the numbers that you are asked to find. Calculations must be shown. (7 marks)



Solution:

Min = 4.0

Sorted data: 4, 4.1, 4.1, 4.2, 4.2, 4.4, 4.5, 4.5, 5.2, 5.2

Median = (4.2+4.4)/2 = 4.3

1st quartile = 25th percentile

$n = 10$, $p = 0.25$, $(n+1)p = 2.75$, $k = 2$, $d = 0.75$

$x_2 = 4.1$, $x_3 = 4.1$

$Q_{0.25} = 4.1 + 0.75 \cdot (4.1-4.1) = 4.1$

3rd quartile = 75th percentile

$n = 10$, $p = 0.75$, $(n+1)p = 8.25$, $k = 8$, $d = 0.25$

$x_8 = 4.5$, $x_9 = 5.2$

$Q_{0.75} = 4.5 + 0.25 \cdot (5.2-4.5) = 4.675$

IQR = $Q_{0.75} - Q_{0.25} = 4.675 - 4.1 = 0.575$

1.5 IQR = 0.8625

4.1 - 0.8625 = 3.2375

Min > 3.2375. Therefore there is no outlier in the lower tail of dataset

2. The time that it takes to complete a marathon is modelled by a normal distribution with a mean of 4.547 hours and a standard deviation of 0.634 hours.

   a) What is the probability that a randomly selected runner will complete the marathon in less than 3 hours and a half? (4 marks)

Solution:

Let X represent the time (in hours) required to complete the marathon

$\mu = 4.547$ hr $\sigma = 0.634$ hr

$$P(X < 3.5) = P\left(Z < \frac{3.5-4.547}{0.634}\right) = P(Z < -1.65) = 0.0495 \approx 0.05$$

The probability that a randomly selected runner will complete the marathon in less than 3.5 hours is approximately **5%**

b) What is the median finish time in this marathon? (4 marks)

Solution:

$P(X < x_o) = P(Z < z_o) = 0.5$

from table $Z_{0.5} = 0$

$.z_o = 0 = \frac{x_o - \mu}{\sigma}$

$x_o = \mu = 4.547\ hr$

The median finish time in this marathon is 4.547 hours (4 hours, 32 minutes, and 49 seconds)

Alternative solution:

Since distribution is normal, median = mean = 4.547 hrs

x_o = 4.547 hr

The median finish time in this marathon is 4.547 hours (4 hours, 32 minutes, and 49 seconds)

c) If 20,000 people participate in 2020 Marathon in Ottawa, then what would be the longest finish time for the top 50 runners? (4 marks)

Solution:

50/20000 = 0.0025

$P(X < x_o) = 0.0025$ $P(Z < z_o) = 0.0025$

from table $Z_{0.0025}$ = -2.81

$.z_o = \frac{x_o - \mu}{\sigma}$

$x_o = \mu + z_o.\sigma = 4.547 - 2.81 \times 0.634\ hr \approx$ 2.765 hr

The top 50 runners finish before or at most in 2.765 hours (2 hours, 45 minutes, 56 second)

3. Two new product designs are to be compared on the basis of revenue potential. Marketing believes that the revenue from design A can be predicted quite accurately to be $3 million. The revenue potential of design B is more difficult to assess. Marketing concludes that there is a probability of 0.4 that the revenue from design B will be $5 million and 0.3 probability that the revenue will be only $3 million. But there are also risks of no revenue or even $1 million loss from this design with probabilities of 0.2 and 0.1, respectively. Which design would you choose? Show all your calculations and clearly explain the reason for your choice of design. (8 marks)

Solution:

$E(X_A)$ = \$3 million

$E(X_B) = \sum x.f(x)$ = \$5(0.4) + \$3(0.3) + \$0(0.2) -\$1 (0.1) = \$2.8 million

$E(X_A) > E(X_B)$

The expected revenue of design A is slightly higher than of design B.

To make a final conclusion, we should also look at the variability (variance or standard deviation) of the revenues of both designs.

VAR $(X_A) = \sigma_A = 0$

There is no variability and therefore no risks involved with design A.

VAR (X_B) =

$\sum_x (x-\mu)^2 f(x) = (5 - 2.8)^2(0.4) + (3 - 2.8)^2(0.3) + (0 - 2.8)^2(0.2) + (-1 - 2.8)^2(0.1) =$

4.96 millions of dollars squared

$\sigma_B = \sqrt{4.96}$ = \$2.23 million

Therefore, design A is definitely the better option.

(Since design A is not variable, it is not necessary to calculate VAR(B), as it is obvious that it will be more variable than design A. But, it is important to mention the comparison.)