Last Name [redacted]	First Name [redacted]	ID [redacted]

ELEC 2607 **Assignment 1** 20 Marks

1. Using the method of exhaustive proof show whether $\overline{a+b+c}$ is equal to $\overline{(a+b)} \cdot \bar{c}$.

2 marks

ABC	A+B	$\overline{A+B}$	$(\overline{A+B}) \cdot C$	$(\overline{A+B}) \cdot \bar{C}$	A+B+C	$\overline{A+B+C}$
000	0	1	0	1	0	1
001	0	1	1	0	1	0
011	1	0	0	0	1	0
010	1	0	0	0	1	0
100	1	0	0	0	1	0
101	1	0	0	0	1	0
111	1	0	0	0	1	0
110	1	0	0	0	1	0
				RHS		LHS

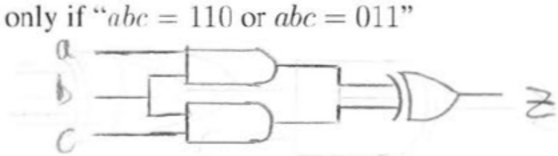
$\therefore \overline{a+b+c} = (\overline{a+b}) \cdot \bar{c}$

2. Draw the simplest logic gate or circuit for the following descriptions. The inputs are a, b, c. The output is z. 4 marks

(a) $z = 1$ only if $abc = 110$

(b) $z = 0$ unless $abc = 010$

(c) $z = 0$ only if "$abc = 110$ or $abc = 011$"



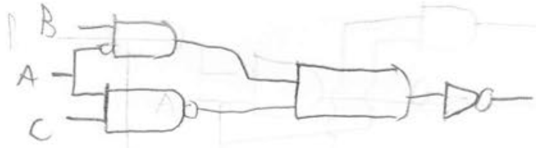
3. Simplify the following 4 marks

(a) $(X + Y + \bar{Y}Z)(Y + X)(Y + \bar{X})$ to 1 literal

$= (x + Y + \bar{Y}z)(x\bar{x} + Y)$
$= (x + Y + \bar{Y}z)(0 + Y)$
$= (x + Y + \bar{Y}z)Y$
$= xY + Y + (0)z$
$= xY + Y$
$= (x+1)Y$
$= Y$

(b) $(BCD + C)CD$ to 2 literals

$= BCD + CD$
$= CD(B+1)$
$= CD$



$\overline{A\bar{B}} = \bar{A}B$

(c) $A\bar{B} + AC + \bar{C}B$ to 3 literals

$= A\bar{B}(C+\bar{C}) + AC + \bar{C}B$

$= A\bar{B}C + A\bar{B}\bar{C} + AC + \bar{C}B$

$= AC(\bar{B}+1) + \bar{C}(A\bar{B}+B)$

$= AC + \bar{C}(A+B)$

$\rightarrow = AC + A\bar{C} + B\bar{C}$

$= A(C+\bar{C}) + B\bar{C}$

$= A + B\bar{C}$

4. Prove algebraically that $(a+b)(b+c)(c+a) = ab + bc + ca$ 2 mark

$(a+b)(b+c)(c+a)$

$= (ab + ac + b + bc)(c+a)$

$= (cab + \underline{ac} + cb + bc + ab + \underline{ac} + ab + abc)$

$= ac + abc + bc + ab$

$= ab + bc + ca + abc$

$= ab + bc + ca + (ab \cdot c) \rightsquigarrow = ab + bc + ca$ //

$ab + (ab \cdot c) = ab$

5. Use duality on $\bar{a}\bar{b} + ab = (a+\bar{b})(\bar{a}+b)$ to find an alternate expression for $\bar{a}b + a\bar{b}$.

1 mark

$\bar{a}\bar{b} + ab = (a+\bar{b})(\bar{a}+b)$

$\hookrightarrow (\bar{a}\bar{b}) \cdot (ab) = (a\bar{b})(\bar{a}+b)$

$\hookrightarrow (\bar{a}+\bar{b})(a+b) = a\bar{b} + \bar{a}b$ //

6. Convert the following expressions to a form where the bar is over single variables only.

4 marks

(a) $\overline{a(b\bar{c}+de) + \overline{(d+a)}cg}$

$= \overline{a(b\bar{c}+de)}\;\overline{\overline{(d+a)}cg}$

$= [\bar{a} + \overline{(b\bar{c}+de)}][(d+a) + \overline{cg}]$

$= [\bar{a} + (\overline{b\bar{c}} \cdot \overline{de})][d + a + \bar{c} + \bar{g}]$

$\rightarrow = [\bar{a} + (\bar{b}+c)(\bar{d}+\bar{e})][d + a + \bar{c} + \bar{g}]$

(b) $\overline{a(b\bar{c} + \bar{e}d) + \overline{\overline{(d+ab)}\;\overline{(cg)}}} + a\overline{dcg}$

$= \overline{a(b\bar{c} + e + \bar{d}) + \overline{(\bar{d} \cdot \overline{ab})(\bar{c}+\bar{g})}} + a(\bar{d} + \overline{cg})$

$= \overline{a(b\bar{c} + e + \bar{d}) + [\overline{\bar{d} \cdot \overline{ab}} + \overline{(\bar{c}+\bar{g})}]} + a(\bar{d} + \bar{c} + \bar{g})$

$= \overline{a(b\bar{c} + e + \bar{d}) + [(d+ab) + cg]} + a(\bar{d} + \bar{c} + \bar{g})$

$= \overline{a(b\bar{c} + e + \bar{d})} \cdot \overline{[(d+ab) + cg]} + a(\bar{d} + \bar{c} + \bar{g})$

$= [\bar{a} + \overline{(b\bar{c} + \bar{e}\bar{d})}] \cdot [\overline{(d+ab)} \cdot \overline{cg}] + a(\bar{d} + \bar{c} + \bar{g})$

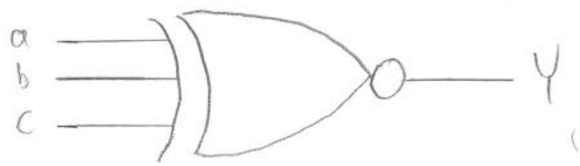
$= [\bar{a} + \overline{b\bar{c}} \cdot \bar{e}d] \cdot [\bar{d} \cdot \overline{ab} \cdot (2\bar{c} + \bar{g})] + a(\bar{d} + \bar{c} + \bar{g})$

$= [\bar{a} + (\bar{b}+c)\bar{e}d] \cdot [\bar{d} \cdot (\bar{a}+\bar{b}) \cdot (\bar{c}+\bar{g})] + a(\bar{d} + \bar{c} + \bar{g})$

7. Derive the equations for y and z from the following table and simplify the answer as much as possible. Then implement y and z using a minimum number of 2-input and 3-input gates (note: inverters (bubbles) don't count). Also identify the function obtained by y and z (eg. is the function an 'and', 'xor', 'or', 'majority gate' etc.?) 3 marks

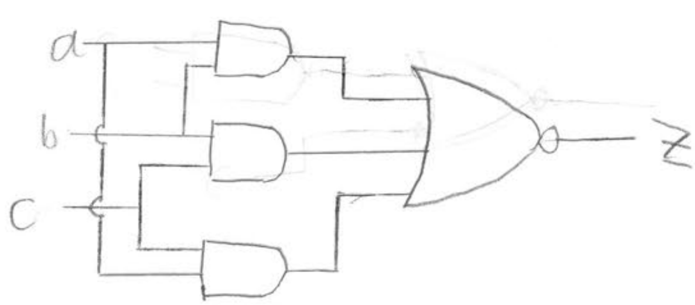
$y = \overline{a \oplus b \oplus c}$ "Even parity"

	c	b	a	y	z
x	0	0	0	1	1
x	0	0	1	0	1
x	0	1	0	0	1
	0	1	1	1	0
x	1	0	0	0	1
	1	0	1	1	0
	1	1	0	1	0
	1	1	1	0	0



$z = \overline{ab + bc + ca}$ "Inverse Majority Input"

c b a	ab	bc	ca	$\overline{ab+bc+ca}$
0 0 0	0	0	0	1
0 0 1	0	0	0	1
0 1 0	0	0	0	1
0 1 1	1	0	0	0
1 0 0	0	0	0	1
1 0 1	0	0	1	0
1 1 0	0	1	0	0
1 1 1	1	1	1	0



$z = \overline{ab + bc + ca}$