CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Duration**
Midterm Test	27 October 2019	1 h 30 min
Special Instructions:	Only approved calculators are allowed **Show your work for full marks**	

1. (12 marks): **(a)** Solve for x (find the *exact* values, do not approximate):

$\log_4(x+3) - \log_4(x-3) = -1$

(b) Let $f(x) = \sqrt{1+x}$ and $g(x) = 2^{1-x^2}$. Find the composite function $g \circ f$ and determine its domain and its range.

(c) Let $f(x) = 5^{x^2}$ and $g(x) = 5^{2x} + 1$. Determine which of these functions is NOT invertible and which one IS (**explain**), and find the inverse of the invertible function.

2. (4 marks) Find the limit or explain why the limit does not exist:

$$\lim_{x \to 3^-} \frac{|\,x-3\,|}{x^2 + x - 12}$$

3. (6 marks) Find **(a)** all horizontal and **(b)** all vertical asymptotes of the graph

$$y = \frac{|x|\,\sqrt{4x^4 + 6x^2 + 4}}{(2x-1)^2\,(x+1)}$$

4. (4 marks) Consider the following piecewise-defined function:

$$f(x) = \begin{cases} e^{a\cdot x} & \text{if} \quad x \le -1 \\ 2\,x + b & \text{if} \quad -1 < x < 1 \\ 6 - x & \text{if} \quad x \ge 1 \end{cases}$$

For what values of a and b is the function f continuous at every x? Sketch the graph of $f\,(x)$.

(continued on the other side)

5. (12 marks) Find the derivatives of the following functions (you don't need to simplify the final answer, but you must show how you calculate it):

(**a**) $f(x) = \dfrac{4x^7 - x^{3/5} + 2\sqrt{x}}{\sqrt[3]{x}}$

(**b**) $f(x) = (x^2 + 2x)\, e^{x^3} + e^3$

(**c**) $f(x) = e^{(e^{\cos x} + x \tan x)}$

6. (4 marks) Find the second derivative of the function $f(x) = \dfrac{\sin x}{1 + \cos x}$, and calculate its value at $x = 0$, i.e. $f''(0)$.

(HINT: simplify the first derivative $f'(x)$ before calculating $f''(x)$.)

7. (8 marks) Given the function $f(x) = \sqrt{3x + 4}$,

(a) Use the definition of the derivative to find the derivative $f'(x)$.

(b) Write equation of the tangent line to the curve $y = f(x)$ at $x = 4$.

Bonus Question (3 marks). Assume that $h(x) = f(g(x))$ where both $f(x)$ and $g(x)$ are defined and are differentiable for all real x. Given $g(-1) = 2$, $g'(-1) = 3$ and $f'(2) = -4$, find the value of $h'(-1)$.