MAT1308 DGD # 6.Fall 2020, October 20

Tangent line

Find the equation of the line tangent to $3x^2 + y^2 = 12$ at the point $(-1, -3)$.

$$\begin{aligned} 6x + 2y\frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= -\frac{3x}{y} \quad \text{plug the value} \\ \frac{dy}{dx} &= -\frac{-3}{-3} = -1 \end{aligned}$$

the tangent line is $y - (-3) = -1(x - (-1))$.
or $y = -x - 4$.

Higher-order derivatives

Find $f'(x), f''(x)$, possible try to find $f^{(n)}(x)$.

a. $f(x) = \dfrac{3}{2x+5}$.

We try to write $f(x)$ as $f(x) = 3(2x+5)^{-1}$, so we can find derivative : $f'(x) = 3(-1)2(2x+5)^{-2}$.
$f''(x) = 3(-1)(-2)2^2(2x+5)^{-3}$. Continue ,
Finally: $f^{(n)}(x) = 3(-1)(-2)\cdots(-n)2^n(2x+5)^{-(n+1)}$.

b. $f(x) = \sqrt{3-2x}$.

$$f'(x) = \frac{1}{2}(3-2x)^{-\frac{1}{2}}(-2)$$

$$f''(x) = \frac{1}{2}\left(-\frac{1}{2}\right)(3-2x)^{-\frac{3}{2}}(-2)^2$$

Critical points

Find the critical points $f(x) = (x^2 - 4x + 3)^3$.

Solution: $f'(x) = 3(x^2 - 4x + 3)^2(2x - 4) = 6\Big((x-3)(x-1)\Big)^2(x-2)$. So the critical values are $x = 1, x = 2, x = 3$.

Increase and decrease

Determine following given functions at

- The interval(s) of x for which $f(x)$ increases;
- The interval(s) of x for which $f(x)$ decreases;
- The extreme points.

$$f(x) = \frac{-2x^3}{x^3 - 12x}$$

$$f'(x) = \frac{-6x^2(x^3 - 12x) - (3x^2 - 12)(-2x^3)}{(x^3 - 12x)^2} = \frac{48x^3}{(x^3 - 12x)^2}.$$ So

$f(x)$ is decreasing on $(-\infty,\ 0)$ and increasing on $(0,\ +\infty)$. At $x = 0$ has extreme point (0.0).

$$f(x) = \frac{3x^2}{x^2 + 25}$$

Make some change about function: $f(x) = \dfrac{3x^2 + 75 - 75}{x^2 + 25} = 3 - \dfrac{75}{x^2 + 25}$.

$$f'(x) = -\frac{75(-2x)}{(x^2 + 25)^2} = \frac{150x}{(x^2 + 25)^2}.$$

$f(x)$ is decreasing on $(-\infty,\ 0)$ and increasing on $(0,\ +\infty)$. At $x = 0$ has extreme point (0.0).

Using the graphing strategy to analyze and sketch the graph of the functions

a. $f(x) = \dfrac{2x^2}{x^2 - 9}$

b. Sketch the graph of a function that satisfies all of the given conditions:

$f'(0) = f'(2) = f'(4) = 0$,

$f'(x) > 0$ if $x < 0$ or $2 < x < 4$, (increasing)

$f'(x) < 0$ if $0 < x < 2$ or $x > 4$, (decreasing)

$f''(x) > 0$ if $1 < x < 3$, $f''(x) < 0$ if $x < 1$ or $x > 3$. (which means on interval $(1,\ 3)$ concave up, others concave down.