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Assignment 1

Question 1

$$f(t) = te^{-5t} \Rightarrow \mathcal{L}\{te^{-5t}\}$$

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt = \int_0^\infty t e^{-5t} e^{-st} dt = \int_0^\infty t e^{-t(5+s)} dt$$

$u = t \quad v =$

$du = 1 \quad dv = e^{-t(5+s)}$

$$\therefore F(s) = \frac{te^{-t(5+s)}}{-(5+s)} - \int 1 \cdot \frac{e^{-t(5+s)}}{-(5+s)} dt$$

$$F(s) = \frac{te^{-t(5+s)}}{-(5+s)} - \frac{e^{-t(5+s)}}{(5+s)^2}\Bigg|_0^\infty$$

$$\boxed{\therefore F(s) = \frac{1}{(s+2)^2}}$$

Question 2

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{10}{s(s+2)(s+3)^2}\right\} \Rightarrow \frac{10}{s(s+2)(s+3)^2} = \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{k_3}{(s+3)^2} + \frac{k_4}{s+3}$$

$$k_1 = \lim_{s \to 0} \frac{10}{(s+2)(s+3)^2} = 5/9$$

$$k_2 = \lim_{s \to -2} \frac{10}{s(s+3)^2} = -5$$

$$k_3 = \lim_{s \to -3} \frac{10}{s(s+2)} = 10/3$$

$$k_4 = \lim_{s \to -3} \left(\frac{10}{s(s+2)}\right)' = \lim_{s \to -3} \frac{-(20)(s+2)}{s^2(s+2)^2} = 40/9$$

$$F(s) = \frac{5}{9}\left(\frac{1}{s}\right) - \frac{5}{s+2} + \frac{10}{3}\left(\frac{1}{(s+3)^2}\right) + \frac{40}{9}\left(\frac{1}{s+3}\right)$$

$$y(t) = \underbrace{\frac{5}{9}u(t)}_{\text{table 2.1.2}} - \underbrace{5\left(e^{-2t}u(t)\right)}_{\text{table 2.1.5}} + \underbrace{\frac{10}{3}\left(te^{-3t}\right)}_{\text{Question 1}} + \underbrace{\frac{40}{9}\left(e^{-3t}u(t)\right)}_{\text{table 2.1.5}}$$

$$y(t) = u(t)\left(\frac{5}{9} - 5e^{-2t} + \frac{10}{3}te^{-3t} + \frac{40}{9}e^{-3t}\right)$$

Question 3

$y(t) = u(t)$, using table 2.2.1, $\mathcal{L}\{u(t)\} = \int_0^\infty u(t) e^{-st} dt$

a/ $\mathcal{L}\{u(t)\} = \frac{e^{-st}}{s}\Big|_0^\infty = \frac{1}{s}$

b/ $\sin(wt)u(t)$, using table 2.2.1, $\mathcal{L}\{\sin(wt)u(t)\} = \int_0^\infty f(t)e^{-st} dt$

$\sin(wt) = (e^{jwt} - e^{-jwt})\frac{1}{2j}$

$\sin(wt)u(t) = [\frac{1}{2j} e^{jwt} - \frac{1}{2j} e^{-jwt}] u(t)$

$\mathcal{L}\{\sin(wt)u(t)\} = \frac{1}{2j}\mathcal{L}\{e^{jwt} - e^{-jwt}\}u(t)$

$= \frac{1}{2j}\left[(\frac{1}{s-jw}) - (\frac{1}{s+jw})\right] = \frac{1}{2j}\left[\frac{s+jw-s+jw}{s^2+w^2}\right]$

$= \frac{1}{2j}\left[\frac{2jw}{s^2+w^2}\right] = \frac{w}{s^2+w^2}$

Question 4

$\mathcal{L}\{e^{-at}\cos(wt)u(t)\} = \int_0^\infty f(t) e^{-st} dt$ (table 2.2.1)

$f(t) = \cos w(t)u(t)$ $\therefore \mathcal{L}\{e^{-at}f(t)\} = F(s+a)$

$\therefore \int_0^\infty \cos(wt)u(t)e^{-st}dt = \frac{s}{s^2+w^2}$ (table 2.1.7)

$\therefore \mathcal{L}\{e^{-at}\cos(wt)u(t)\} = \frac{s+a}{(s+a)^2+w^2}$

Question 5

$x'' + 2x' + 2x = \sin(2t)$ (table 2.1.6, 2.2.3, 2.2.7)

$s^2F(s) - sX(0) - x'(0) + 2(sF(s) - x(0)) + 2F(s) = \frac{2}{s^2+4}$

$(s^2+2s+2)F(s) = \frac{2}{s^2+4} + 4s + 4$

$$F(s) = \underbrace{\frac{2}{(s^2+4)(s^2+2s+2)}}_{\text{Part 2}} + \underbrace{\frac{4s+4}{(s^2+2s+2)}}_{\text{part 1}}$$

part 1:

$$\frac{4s+4}{s^2+2s+2} = \frac{4(s+1)}{(s+1)^2+1}$$ (using steps in Question 4 and table 2.1.7)

$$\mathcal{L}^{-1}\left\{\frac{4(s+1)}{(s+1)^2+1}\right\} = 4e^{-t}\cos(t)$$

Part 2:

$$\frac{2}{(s^2+4)(s^2+2s+2)} = \frac{k_1 s+k_2}{s^2+4} + \frac{k_3 s+k_4}{s^2+2s+2}$$

$\therefore$ $2 = (k_1 s + k_2)(s^2+2s+2) + (k_3 s + k_4)(s^2+4)$

$2 = k_1 s^3 + 2k_1 s^2 + 2k_1 s + k_2 s^2 + 2k_2 s + 2k_2 + k_3 s^3 + 4k_3 s + k_4 s^2 + 4k_4$

$2 = (k_1 + k_3)s^3 + (2k_1 + k_2 + k_4)s^2 + (2k_1 + 2k_2 + 4k_3)s + 2k_2 + 4k_4$

$\therefore$ $k_1 + k_3 = 0$ $\qquad$ $k_4 = 3/5$

$2k_1 + k_2 + k_4 = 0$ $\qquad$ $k_3 = 1/5$

$2k_1 + 2k_2 + 4k_3 = 0$ $\rightarrow$ $k_2 = -1/5$

$2k_2 + 4k_4 = 2$ $\qquad$ $k_1 = -1/5$

$$\therefore \frac{-1}{5}\cdot\frac{s+1}{s^2+4} + \frac{1}{5}\cdot\frac{s+3}{s^2+2s+2} = \frac{-1}{5}\cdot\frac{s}{s^2+4} - \frac{1}{5}\cdot\frac{1}{s^2+4} + \frac{1}{5}\cdot\frac{s+1+2}{(s+1)^2+1}$$

$$= \frac{-1}{5}\cdot\frac{s}{s^2+4} - \frac{1}{5}\left(\frac{1}{s^2+4}\right) + \frac{1}{5}\left(\frac{s+1}{(s+1)^2+1}\right) + \frac{2}{5}\left(\frac{1}{(s+1)^2+1}\right)$$

using table 2.1

Part 2: $y(t) = \frac{-1}{5}\cos(2t) - \frac{1}{5}\left(\frac{1}{2}\right)\sin(2t) + \frac{1}{5}e^{-t}\cos(t) + \frac{2}{5}e^{-t}\sin t$

part 1 + part 2

$$y(t) = \frac{-1}{5}\cos(2t) - \frac{1}{10}\sin(2t) + \frac{1}{5}e^{-t}\cos(t) + \frac{2}{5}e^{-t}\sin(t) + 4e^{-t}\cos(t)$$