This test paper has two parts and total of 44 marks.

Part I has 6 multiple choice questions. Part II has 3 long answer questions.

It cannot be taken from the examination room.

Only nonprogrammable calculators are allowed. Duration: 50 Minutes.

NAME : **STUDENT NO :**

PART I: Multiple Choice Questions. No partial marks. Circle the correct answer.

[3] **1)** If A is a 3×3 matrix that is not invertible, then which of the following could be the characteristic polynomial of A?

a) $\lambda^3 - 1$ **b)** $\lambda(\lambda^3 + 1)$ **c)** $\lambda^3 + 2\lambda^2 + 3$ **d)** $\lambda^3 - \lambda$

[3] **2)** Let $A = \begin{bmatrix} 3 & 1 & 2 & 0 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$.

What is the dimension of the eigenspace of A corresponding to the eigenvalue $\lambda = 3$?

a) 1 **b)** 2 **c)** 3 **d)** 4

[3] **3)** Let $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$. What are the eigenvalues of A?

a) ± 1 **b)** $1 \pm i$ **c)** $-1 \pm i$ **d)** $\pm i$

[3] **4)** Let $A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$.

You are given that v is an eigenvector of A. What is the corresponding eigenvalue?

a) -2 **b)** -1 **c)** 3 **d)** 4

[3] **5)** What is the standard form of the complex number $\frac{1-5i}{2+i}$?

a) $\frac{11}{5} - \frac{3}{5}i$ **b)** $\frac{-3}{5} - \frac{11}{5}i$ **c)** $\frac{3}{5} + \frac{11}{5}i$ **d)** $\frac{-11}{5} + \frac{3}{5}i$

[3] **6)** Find all the (complex) solutions to the equation $z^3 = 1$?

a) 1 **b)** $1,\ i,\ -i$ **c)** $1,\ -1+\sqrt{3}i,\ -1-\sqrt{3}i$ **d)** $1,\ -\frac{1}{2}+\frac{\sqrt{3}}{2}i,\ -\frac{1}{2}-\frac{\sqrt{3}}{2}i$

PART II: Long answer questions. Show all your work.

[**12**] **1)** Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. You are given that $\det(A - \lambda I) = -(\lambda - 1)^2(\lambda - 3)$.

a) Find the eigenvalues of A.
b) Find a basis for each eigenspace of A.
c) Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

[10] 2) Let $A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$.

Find an invertible matrix P and a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ such that $A = PCP^{-1}$.

[4] 3) Let A be a 2×2 matrix with the eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 0$ and corresponding eigenvectors $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, respectively.

(a) Find the matrix A,

(b) Find the matrix A^{2007}.