Online Homework System

Assignment Worksheet
9/21/21 - 9:41:45 PM EDT

Name: ______________________________

Class: MAT1320[B] Calculus I 20219

Class #: ______________________________

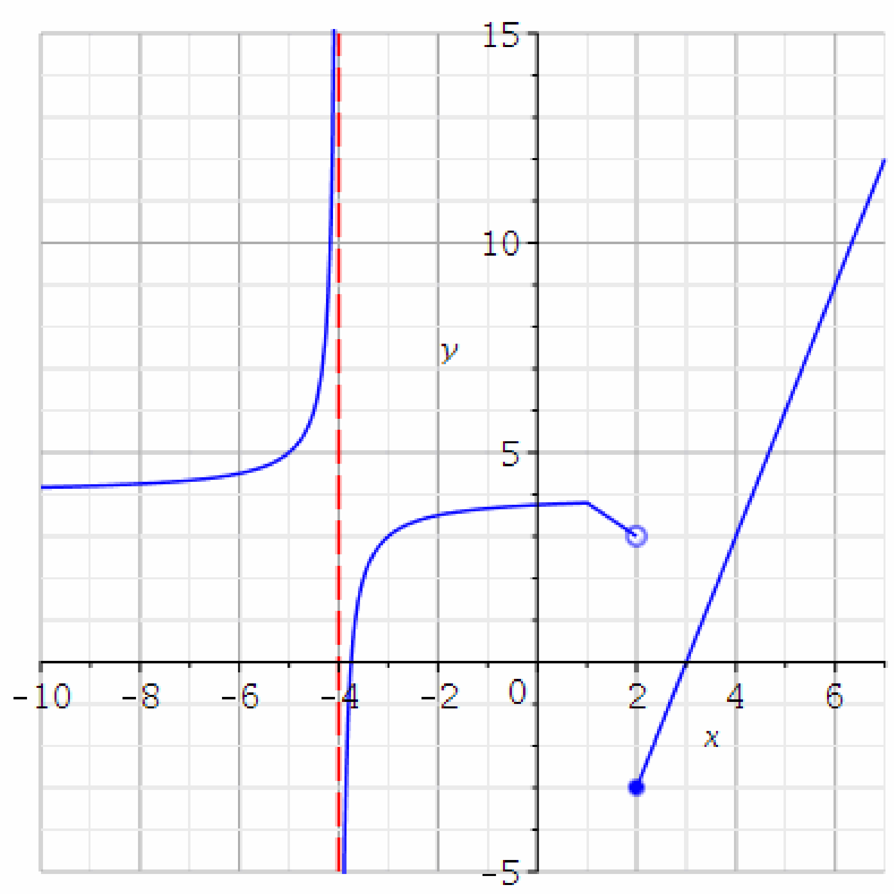
Section #: ______________________________

Instructor: Elizabeth Jane Maltais

Assignment: Assignment 2

Question 1: (1 point)

Consider the graph of the function f below. The dashed line represents a vertical asymptote. The circles are points with integer coordinates.



a) Give the left and right limits of this functions at the various points specified. Write "inf" for ∞ and "-inf" for $-\infty$.

$\lim_{x \to -4^-} f =$ __________

$\lim_{x \to -4^+} f =$ __________

$\lim_{x \to 2^-} f =$ __________

$\lim_{x \to 2^+} f =$ __________

b) At which of the following values of x does the limit of f exist? The choices are $-8; -4; 0, 2; 4$.

Answer: __________

FORMATTING: *If more than one of these points is correct, separate your answers with a semi-colon (;).*

Question 2: (1 point)

Use your knowledge of these functions to identify the following limits:

FORMATTING: *For this question, if the limit diverges to* ∞*, write* ***infty****; if it diverges to* $-\infty$*, write* ***-infty****.*

a) $\lim_{x \to -\infty} e^x =$__________

b) $\lim_{x \to \infty} e^x =$__________

c) $\lim_{x \to 0^+} \ln(x) =$__________

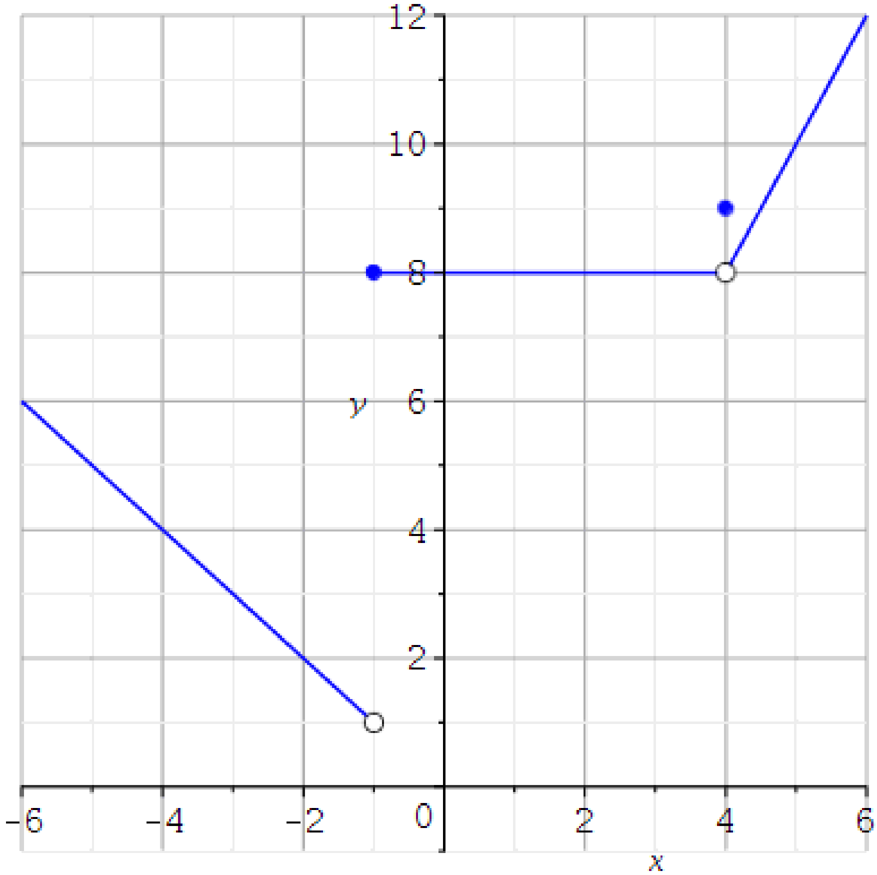
d) $\lim_{x \to 0^+} \frac{1}{x} =$__________

e) $\lim_{x \to \infty} \frac{1}{x} =$__________ .

f) $\lim_{x \to 0^-} \frac{1}{x} =$__________

Question 3: (1 point)

The graph of a function $g(x)$ is given below. (All interesting features have integer coordinates.)



a) List the points $x = a$ at which $\lim_{x \to a} g(x)$ does not exist.

$a =$__________.

FORMATTING: *If there is more than one solution, separate them with a semicolon(**;**). If there is no such point, type **empty**.*

b) List the points $x = b$ at which g is discontinuous.

$b =$__________.

FORMATTING: *If there is more than one solution, separate them with a semicolon(**;**). If there is no such point, type **empty**.*

Question 4: (1 point)

We want to compute the following limit.

$$\lim_{t\to 5}\left(\frac{1}{t-5}-\frac{10}{t^2-25}\right)$$

a) As t approaches 5 this gives an indeterminate form of the type

$\infty - \infty$

[ ]

$0 \times \infty$

[ ]

0^0

[ ]

∞/∞

[ ]

1^∞

[ ]

$0/0$

[ ]

b) When we put these fractions over a common denominator, we can simplify the expression so that it takes the form

$$\left(\frac{1}{t-5}-\frac{10}{t^2-25}\right)=\frac{1}{q(t)},$$

where $q(t)$ is a polynomial.

Answer: $q(t) =$ __________

c) Compute $\lim_{t\to 5}\left(\frac{1}{t-5}-\frac{10}{t^2-25}\right)$ using your result from part (b).

Answer: __________

Question 5: (2 points)

Consider the function f defined by $f(x)=\frac{4+3e^{-5x}}{5-16e^{-5x}}$. Our goal in this question is to understand its behaviour as x goes to $\pm\infty$, as well as near gaps in its domain.

a) First compute $\lim_{x\to\infty} f(x)$. Answer: __________

b) Next we want to compute the limit in the opposite direction. As x goes to $-\infty$, we get an indeterminate form of the type

$0/0$

[ ]

1^{∞}

[ ]

∞/∞

[ ]

0^0

[ ]

$0 \times \infty$

[ ]

$\infty - \infty$

[ ]

c) To be able to compute $\lim_{x \to -\infty} f(x)$, you have to rewrite $f(x)$ as a fraction $f(x) = \frac{P(x)}{Q(x)}$, which is no longer the indeterminate form that you have found in (b). Find $P(x)$ and $Q(x)$.

FORMATTING: *Write your answer in the form* $[P(x), Q(x)]$, *including the square brackets and comma.* ***Strict*** *calculator notation is required in your answer, meaning* **** for multiplication*** *(e.g.* $3x$ *is written* **3*x**, *and* e^{3x} *is written* **e^(3*x)**).

Answer: $[P(x), Q(x)] =$ __________

d) Using your answer in **(c)**, compute $\lim_{x \to -\infty} f(x)$. Answer: __________

Finally, we analyse $f(x)$ near gaps in its domain.

e) Find the point(s) $x = a$ where $f(a)$ is undefined.

FORMATTING: *List these points. If there are two or more such points, you must separate them by semi-colons (;). Since we are asking for exact values, your answers may involve the logarithm function* **ln()**.

Answer: __________

f) For each of the values of \)a\) that you have found in (e), find the left-hand and right-hand limits of f as x approaches a. Then answer the questions below.

FORMATTING: *If there are two or more points, separate them with semi-colons (;). Since we are asking for exact values, your answers may involve the logarithmic function* **ln()**). *You must use the notation of scientific calculators So* $2x$ *is written* ***2 * x***, *and so on. If there aren't any points, write* ***empty***.

For which point(s) a *do we have* $\lim_{x \to a^-} f(x) = +\infty$? *Answer:* __________

For which point(s) a *do we have* $\lim_{x \to a^-} f(x) = -\infty$? *Answer:* __________

For which point(s) a *do we have* $\lim_{x \to a^+} f(x) = +\infty$? *Answer:* __________

For which point(s) a *do we have* $\lim_{x \to a^+} f(x) = -\infty$? *Answer:* __________

Question 6: (1 point)

We want to compute

$$\lim_{x\to\infty} \frac{8x^2}{\sqrt{7x^4+7}}$$

.

a) At x approaches ∞, we get an indeterminate form of the type

$\infty - \infty$

[ ]

$0/0$

[ ]

1^{∞}

[ ]

0^0

[ ]

$0 \times \infty$

[ ]

∞/∞

[ ]

FORMATTING: *In the denominator, note that you are working with a function under a square root, so handle with care. Remember that $\sqrt{x}$ is written as "sqrt(x)" in the response area.*

b) By factoring from the numerator and denominator appropriately, rewrite $\frac{8x^2}{\sqrt{7x^4+7}}$ in the form $\frac{8}{\sqrt{7+g(x)}}$, where

$g(x) =$ __________

c) Compute $\lim_{x\to\infty} g(x)$.

Answer: __________

d) To compute the limit in **(e)**, we must use the fact that $h(y) = \frac{8}{\sqrt{7+y}}$ is continuous at a certain point y.

Answer: $y =$ __________ .

e) Using your results in (c) and (d), compute $\lim_{x\to\infty} \frac{8x^2}{\sqrt{7x^4+7}}$

FORMATTING: *You must give the exact value of the limit, not a numerical approximation.*

Answer: __________

Question 7: (1 point)

Consider the function g defined by $g(x) = \dfrac{x^2 - 9}{|x-3|}$ for $x \neq 3$.

Our goal is to understand the behavior of g near $x = 3$.

a) As x approaches 3 this gives an indeterminate form of the type

0^0

[ ]

1^∞

[ ]

$0 \times \infty$

[ ]

$0/0$

[ ]

$\infty - \infty$

[ ]

∞/∞

[ ]

Suppose first that $x > 3$.

b) If $x > 3$, then $|x-3| =$ __________

FORMATTING: *Your answer should not have an absolute value.*

Therefore, when $x > 3$ we may simplify the expression for $g(x)$ to get

$g(x) =$ __________ .

Hence, $\lim\limits_{x \to 3^+} g(x) =$ __________

Suppose now that $x < 3$.

c) If $x < 3$, then $|x-3| =$ __________

FORMATTING: *Your answer should not have an absolute value.*

Therefore, when $x < 3$ we may simplify the expression for $g(x)$ to get

$g(x) =$ __________ .

Hence, $\lim\limits_{x \to 3^-} g(x) =$ __________

d) Using (b) and (c) we conclude $\lim\limits_{x \to 3} g(x) =$ __________

FORMATTING: *if the limit doesn't exist write **diverges**.*

Question 8: (1 point)

We want to compute the following limit

$$\lim_{t \to 0} \frac{2t}{1-\sqrt{1+7t}}.$$

a) As t approaches 0, this gives an indeterminate form of the type

0^0

[ ]

$0/0$

[ ]

∞/∞

[ ]

1^∞

[ ]

$\infty - \infty$

[ ]

$0 \times \infty$

[ ]

We consider the function f defined by $f(t) = \dfrac{2t}{1-\sqrt{1+7t}}$ for $t \neq 0$.

Since we are faced with an indeterminate form, we need to simplify $f(t)$ before evaluating the limit.

b) Select which strategy you will use to simplify $f(t)$:

(a) Factoring

(b) Rationalization

(c) Partial fraction decomposition

(d) log transformation

c**)** Simplify $f(t)$ using the chosen method on **b)** so that your simplified function isn't an indeterminate form as t approaches 0.

The simplified form of $f(t) =$__________

FORMATTING: *In Mobius, $\sqrt{a}$ is written "sqrt(a)".*

d) Using your simplified form of $f(t)$, compute the limit:

$$\lim_{t \to 0} \frac{2t}{1-\sqrt{1+7t}} = \_\_\_\_\_\_\_\_\_\_$$

Question 9: (1 point)

We want to compute the following limit

$$\lim_{x \to 6} \frac{\sin(2(x-6))}{\sin(x-6)}.$$

a) As x approaches 6 we get an indeterminate form of the type

1^{∞}

[ ]

∞/∞

[ ]

$0 \times \infty$

[ ]

0^0

[ ]

$\infty - \infty$

[ ]

$0/0$

[ ]

b) We want to simplify the quotient $\frac{\sin(2(x-6))}{\sin(x-6)}$ for $x \neq 6$, so that it no longer gives an indeterminate form at $x = 6$.

HINT: *Use trigonometric identities.*

Answer : __________

c) Use your result in (b) to compute $\lim\limits_{x \to 6} \frac{\sin(2(x-6))}{\sin(x-6)}$.

Answer: __________

Question 10: (1 point)

We want to compute the following limit:

$$\lim_{x \to 5} \frac{-x^2 + 7x - 10}{x^2 - 7x + 10}.$$

a) As x approaches 5, we get an indeterminate form of the type

1^∞

[ ]

$0/0$

[ ]

∞/∞

[ ]

$0 \times \infty$

[ ]

$\infty - \infty$

[ ]

0^0

[ ]

b) Simplify the quotient $\frac{-x^2+7x-10}{x^2-7x+10}$ when $x \neq 5,$ to get an expression of the form $\frac{P(x)}{Q(x)}$ that doesn't have a division by 0 at $x = 5$.

FORMATTING: *To write your answer, please type it in the form* $[P(x), Q(x)]$, *including the square brackets and write a comma between your simplified numerator* $P(x)$ *and your simplified denominator* $Q(x)$.

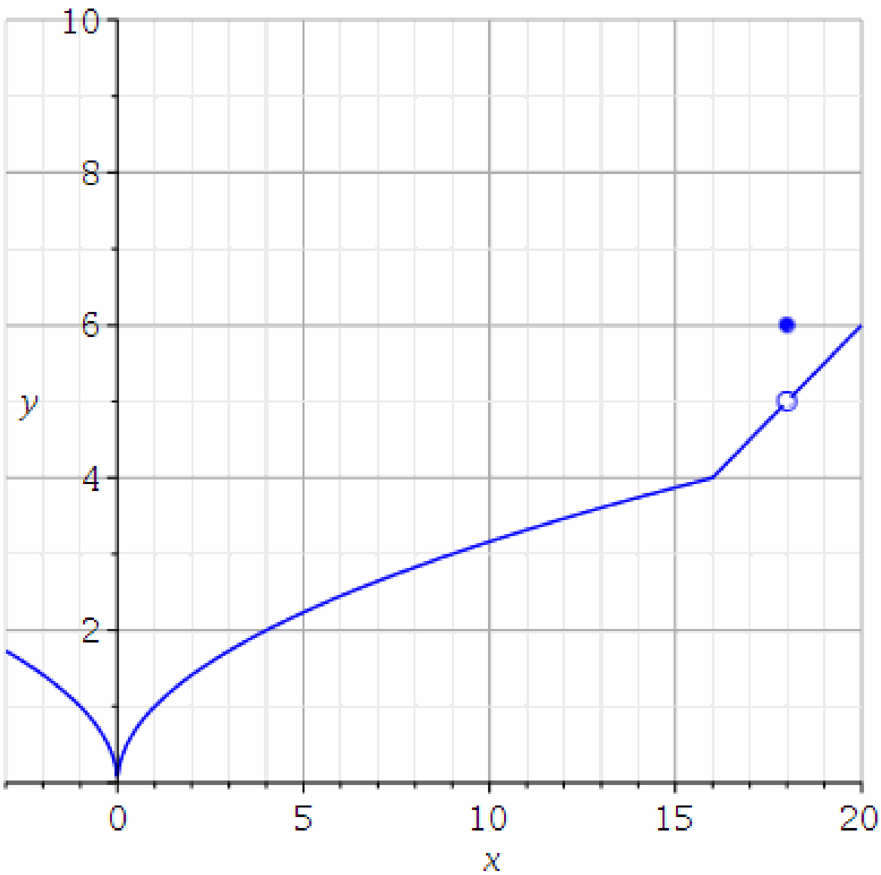
Answer: $[P(x), Q(x)] =$ __________

c) Use your result in (b) to compute $\lim_{x \to 5} \frac{-x^2 + 7x - 10}{x^2 - 7x + 10}$.

Answer: __________

Question 11: (1 point)

Consider the graph of the function f below. All interesting features have integer coordinates.



a) Calculate the left and right limits of these functions at the various points specified. Write **diverges** if the limit does not exist.

$\lim_{x \to 0^-} f(x) =$ __________

$\lim_{x \to 0^+} f(x) =$ __________

$\lim_{x \to 18^-} f(x) =$ __________

$\lim_{x \to 18^+} f(x) =$ __________

b) At which of the points $x = 0; 16; 18; 19$ does the limit of f exist?

Answer: __________

FORMATTING: *If you have more than one answer, separate them with a semi-colon (;).*

c) At which of the points $x = 0; 16; 18; 19$ is the function continuous?

Answer: __________

FORMATTING: *If you have more than one answer, separate them with a semi-colon (;).*

Question 12: (1 point)

For which values of a and b is the following function continuous everywhere?

$$f(x) = \begin{cases} a\sin(\pi x) + b & \text{if } x \le 4 \\ x^2 + a & \text{if } 4 < x \le 5 \\ b\cos(2\pi x) + a & \text{if } x > 5 \end{cases}$$

Answer:

$a =$ __________

$b =$ __________

Question 13: (1 point)

Consider the function

$$f(x) = \begin{cases} \frac{x^2-11x+28}{x-4} & \text{if } x < 4 \\ x + a & \text{if } x \ge 4 \end{cases}$$

a) Simplify the expression for $f(x)$ when $x < 4$.

$\frac{x^2-11x+28}{x-4} =$ __________

b) Compute the following limit.

$\lim_{x \to 4^-} f(x) =$ __________

c) Compute the following limit.

$\lim_{x \to 4^+} f(x) =$ __________

d) Find the value of a such that f is continuous at $x = 4$.

Answer: $a =$ __________

Question 14: (1 point)

Consider the function

$$f(x) = \begin{cases} \frac{|x-3|}{x^2-9} & \text{if } x \neq 3 \\ \frac{1}{x+a} & \text{if } x = 3 \end{cases}$$

a) Suppose first that $x < 3$.

In this case, simplify the following expression so that it does not involve an absolute value:

$\frac{|x-3|}{x^2-9} =$ ________

b) Compute the following limit.

$\lim_{x \to 3^-} f(x) =$ ________

c) Now suppose that $x > 3$.

In this case, simplify the following expression so that it does not involve an absolute value:

$\frac{|x-3|}{x^2-9} =$ ________

d) Compute the following limit.

$\lim_{x \to 3^+} f(x) =$ ________

e) If possible, find the value of a such that f is continuous at $x = 3$. If there is no value, write **empty**.

$a =$ ________