ADM 4351, Fall 2020, Quiz #4

Question 1: Find the price of a 1-year European call option with strike price of X=$95 if the current stock price is $100 and each 6 month it can either increase by 20% or decrease by 10%. The risk-free interest rate is 7%.

Ans: p=(exp(0.07*6/12)-0.9)/(1.2-0.9)=0.4521
Stock prices at the end: 144, 108, 81; option payoffs $49, $13, $0. Hence
Price = (p^2*49+2*p*(1-p)*13+(1-p)^2*0)*exp(-0.07*1)=$15.34

Question 2: Repeat previous question assuming the option is American.

Ans: The answer will be the same since it is never optimal to exercise American call options on non-dividend paying stock before maturity. The same answer can be achieved by computing the option price using backward induction method.

Give full mark either if the student say that the price is the same (even if they incorrectly computed the price in the previous question) or if they computed the correct price here

Question 3: Find the price of a 1-year European put option with strike price of X=$83 if the current stock price is $80 and each 6 month it can either increase or decrease by 10% The risk-free interest rate is 7%.

Ans: p=(exp(0.07*6/12)-0.9)/(1.1-0.9)=0.6781
Stock prices at the end: $96.8, $79.2, $64.8.; option payoffs $0, $3.8, $18.2. Hence
Price = (p^2*0+2*p*(1-p)*3.8+(1-p)^2*18.2)*exp(-0.07*1)=$3.305

Question 4: Repeat previous question assuming the option is American.

Ans: In 6 month the stock prices could be $88 or $72 and immediate exercise would give $0 or $11. In bottom state at 6 month the option price from not exercising is
(p*3.8+(1-p)*18.2)*exp(-0.07*6/12)=$8.15<11, hence, early exercise is better.
In top state at 6 month the value is (p*0+(1-p)*3.8)*exp(-0.07*6/12)=$1.1812
At t=0 immediate exercise gives $3 while waiting gives
(p*1.1812+(1-p)*11)*exp(-0.07*6/12)=$4.19

Question 5: Find the price of a 9-month European call option with strike price of X=$80 if the current stock price is $100 and each 3 months it can either increase or decrease by 10%. The risk-free interest rate is 7%.

Ans: p=(exp(0.07*3/12)-0.9)/(1.1-0.9)=0.5881
Stock prices at the end: 133.1, 108.9, 89.1, 72.9; option payoffs $53.1, $28.9, $9.1, $0. Hence
Price = (p^3*53.1+3*p^2*(1-p)*28.9+3*p*(1-p)^2*9.1+(1-p)^3*0)*exp(-0.07*9/12)=$24.56

Question 6: Find the price of a 6-month European call option on £1 with strike price 2.02$/£ if the current exchange rate is 2$/£, in 6 month it can increase to 2.1$/£ or decrease to 1.8$/£. The U.S. ($) interest rate is 5% and UK (£) interest rate is 8%. Keep at least 4 decimal digits in your answer

Ans: p=(exp((0.05-0.08)*6/12)-1.8/2)/(2.1/2-1.8/2)=0.5674
Price = (p*0.08+(1-p)*0)*exp(-0.05*6/12)=$0.0443

Question 7: Find UK (£) interest rate if U.S. ($) interest rate is 5%, current exchange rate is 2$/£, the price of a 6-month European call option on 1£ with strike price 2.02$/£ is $0.04 and the price of a 6-month European put option on £1 with strike price 2.02$/£ is $0.025. Keep at least 6 decimal digits in your answer.

Ans:
From Call-Put parity:
0.04-0.025=2*exp(-q*6/12)-2.02*exp(-0.05*6/16)
Hence, q=-12/6*ln((0.015+2.02*exp(-0.05*6/16))/2)=1.493%

Question 8: Given the information in the previous question, if the UK interest rate is higher than the one you should have computed, which of the following is true:
A) You cannot make an arbitrage
B) You can make an arbitrage and you arbitrage strategy, among other things, will include buying £ today
C) You can make an arbitrage and you arbitrage strategy, among other things, will include selling £ today
D) You can make an arbitrage but provided information is not sufficient to choose between answers B and C above

Ans: B

Question 9: Find the price of a 6-month European call option with strike price of $105 on 2-year Forward contract on a stock if today the futures price is $100 and each 6 month it can increase by 15% or decrease by 10%. The interest rate is 7%

Ans:
p=(1-0.9)/(1.15-0.9)=0.4
In 6 month Futures price is either $115 or $90 and the option value is $10 or $0. Hence, the option price is
(p*10+(1-p)*0)*exp(-0.07*6/12)=$3.86

Question 10: Assume AAA wants to borrow $1,000,000 at fixed rate and BBB wants to borrow $1,000,000 at floating rate. AAA can borrow either at LIBOR+2% or at 8%, BBB can borrow either at LIBOR+1% or at 6.6%. Find QSD

Ans: 8+L+1-(L+2+6.6)=0.4%

Question 11: Using the information in the previous question, assume AAA and BBB enter into a mutually beneficial swap in which they exchange LIBOR interest rate payments for a fixed x% interest rate payments. Find the maximum and minimum values of x.

Ans:
For AAA: L+2-L+x≤8. Hence, x≤6% (maximum value of x)
For BBB: 6.6+L-x≤L+1. Hence, x≥5.6% (minimum value of x)

Question 12: Find the value of the swap agreement that has 14 month till maturity in which you receive floating payments and pay fixed rate payments based on 6% annual rate on a nominal amount of $1,000,000. All payments are semi-annual (i.e., you pay $1,000,000*0.06/2=$30,000 and you get $1,000,000*LIBOR/2 each 6 month). Find the value of this swap for you if the next payment that you will receive is $35,000 and continuously compounded interest rates for 2, 6, 8, 12, and 14 months are 6.8%, 6.6%, 6.5%, 6.47% and 6.45% respectively

Ans:
Fixed-coupon bond:
30000*exp(-0.068*2/12)+30000*exp(-0.065*8/12)+1030000*exp(-0.0645*14/12)=$1,013,727
Floating-rate bond: 1035000*exp(-0.068*2/12)=1,023,226
The value of the swap: 1,023,226-1,013,727=$9,609 or $9,610 (accept negative too)