

**ADM 4351, Fall 2020, Quiz #1 (15 points total) - Solutions**

**Time allowed: 80 minutes + 10 minutes for submission**

**IMPORTANT:**

- All interest rates are continuously compounded (unless specified otherwise).
- The face value of all bonds is \$100, the coupon payments are semiannual.
- You must keep at least 4 decimal digits while performing your calculations (unless specified otherwise).

**Question 1:**

At 11am on September 1 you bought December futures contract on 1,000 bushels of wheat. At that time the spot price was \$3.25 and the basis was \$-0.11 (note that the basis is negative). The initial margin is \$1,000, and the maintenance margin is \$800. What will be size of your margin account in the morning of September 2, if, at the end of September 1, the spot price was equal to \$3.35 and the basis was equal to \$-0.12.

**Answer:**  $F=S-B$ , hence,  $F1=\$3.36$ ,  $F2=\$3.47$ ;  $won = (3.47-3.36)*1000=\$110$ ;  
 $MA=\$1,110$

**Question 2:** In the morning of Sept 23, 2020 **only** three traders (Alex, Barbara, and Christine) had open position in futures contract for corn with delivery in January 2021. In particular, Alex had a long position in 5 contracts and Barbara had a short position in 13 contracts. Information about the position of Christine is not given to you. During the day of Sept 24, 2020 the following trades occurred:

- Alex sold 6 contracts to Barbara
- Barbara bought 10 contract from Christine
- Diane, a new trader, bought 4 contracts from Christine

What was the open interest at the end of Sept 5 trading day?

**A:**  $5-6=-1$ ; **B:**  $-13+6+10=3$ ; **C:**  $(13-5)-10-4=-6$ ; **D:**  $4$ .

**Open interest** =  $3+4=7$

**Answer: 7**

**Question 3:** You farm expects to produce 1000 tons of organic potatoes next summer. You would like to hedge the risk of uncertain wholesale prices on organic potatoes next summer, but, unfortunately, futures contracts available only on regular (not organic) potatoes. The expected price of organic potatoes is \$500/ton with the standard deviation of \$70; the expected futures price of regular potatoes is \$250/ton with standard deviation of \$50, and the correlation between these two prices is 0.85. The contract size is 1 ton. How many contracts you should sell to hedge your risk?

$0.85*1000*70/50=1190$  contracts

Unfortunately, the posted question had a typo and stated that the standard deviation of the price of organic potatoes is \$700 instead of \$70. In this case the answer would be  $0.85*1000*700/50=11900$ . Give full credit for this answer too.

**Question 4:** Consider the same information as in Question 3, but, regardless of your answer for Question 3, assume you've decided to hedge by selling 1250 contracts. What is the standard deviation of your future revenue with hedge?

$$\text{s.d.}(1000x-1250y)=\sqrt{1000*1000*70*70+1250*1250*50*50-2*0.85*1000*1250*70*50}=\$36,997$$

(approximately \$37,000)

If students used \$700 instead of \$70, the answer will be  
 $\sqrt{1000*1000*700*700+1250*1250*50*50-2*0.85*1000*1250*700*50}=\$ 647,712$   
 Give full credit for this answer too.

**Question 5:** Your portfolio's value is \$5,000,000 and its beta = 1.2. You would like to reduce the beta of you portfolio to make it equal to 0.8 by selling S&P500 index futures. The futures price of S&P500 index futures is 2000 and the contract size is \$250 times the index. How many futures contracts you need to sell to hedge your systematic risk?

$$\text{Sell } (1.2-0.8)*5000000/(2000*250)=4 \text{ futures}$$

**Question 6:** Find continuously compounded interest rate if quarterly compounded APR is equal to 18%. YOU MUST KEEP AT LEAST 6 DECIMAL POINTS WHILE PERFORMING YOUR CALCULATIONS!!!

$$\text{Solution: } \exp(r/4)=(1+0.18/4). \text{ Hence, } r=4*\ln(1+0.18/4)=17.6068\%$$

**Question 7:** Find the price of the 8% coupon bond that matures 16 month from now if zero rates for the next 2 years are given by  $r(t)=0.07+0.01t$ , where  $0<t<2$ ,  $t$  is measured in years can be any real number. Report you answer up to the 4<sup>th</sup> decimal digit

Ans:  
 $104*\exp(-(-0.07+0.01*16/12)*16/12)+4*\exp(-(-0.07+0.01*10/12)*10/12)+4*\exp(-(-0.07+0.01*4/12)*4/12)=101.7139$

**Question 8:** Find 4-month zero if 5% coupon bond that matures 4 month from now is selling for \$101

$$101=102.5*\exp(-r*4/12). \text{ Hence, } r=-\ln(101/102.5)*12/4=4.4227\%$$

**Question 9:** Find 10-month zero rate if 4-month 4% coupon bond is selling for \$101 and 10-month 6% coupon bond is selling for \$104

$$\text{Ans: 4-year rate: } 101=102*\exp(-r*4/12). \text{ Hence, } r=\ln(102/101)*12/4=2.9557\%$$

$$10\text{-year: } 104 = 103 \cdot \exp(-r \cdot 10/12) + 3 \cdot \exp(-0.029557 \cdot 4/12)$$

$$R = -\ln((104 - 3 \cdot \exp(-0.029557 \cdot 4/12)) / 103) \cdot 12/10 = 2.3181\%$$

**Question 10:** Find 7-year zero rate if 7-year 9% coupon bond is selling for \$109 and 7-year 6% coupon bond is selling for \$96

Ans: buy 9 of 6%, sell 6 of 9%, pay  $9 \cdot 96 - 6 \cdot 109 = 210$

$$r = \ln(300/210)/7 = 5.0954\%$$

The question had a typo and read as:

Find 7-year zero rate if 6-year 9% coupon bond is selling for \$109 and 7-year 6% coupon bond is selling for \$96

**For students who did not see the announcement about the typos, accept the following two solutions as correct:**

1)  $r = \ln(300/210)/6 = 5.9446\%$ , or

2) A statement that the provided information is not sufficient to find 7-year zero rate

**Question 11:** Find 8-year zero rate if 5-year zero rate is 4% and forward rate between year 5 and year 8 is 5%

Ans:  $5 = (8 \cdot r - 5 \cdot 4) / (8 - 5)$ , Hence,  $r = (5 \cdot (8 - 5) + 5 \cdot 4) / 8 = 4.375\%$

The question had a typo and read as:

Find 8-year zero rate if 3-year zero rate is 4% and forward rate between year 5 and year 8 is 5%

For students who did not see the announcement about the typos, accept the following three solutions as correct

1)  $5 = (8 \cdot r - 3 \cdot 4) / (8 - 3)$ , Hence,  $r = (5 \cdot (8 - 3) + 3 \cdot 4) / 8 = 4.625\%$  or

2) A statement that the provided information is not sufficient to find 8-year zero rate

**Question 12:** Consider three bonds, AAA, BBB, and CCC, all of them maturing 13 years from now. Bond AAA is a 12% coupon bond with current price of \$115, BBB is a 10%

coupon bond currently selling at par (price = \$100). Bond CCC is an 7% coupon bond. Find the current price of CCC

Unfortunately, the question had a mistake that was not discovered during the quiz, in particular, it also stated that 10-year zero rate is 10.2%. Such 10-year zero rate is inconsistent with bond prices and results in arbitrage. As a result, give 1 point to everybody for this question. The solution for the corrected question would have been as follows.

Ans:

1) Using AAA and BBB, buy 12 of BBB, sell 10 of AAA. Pay  $(12*100-10*115=50)$ , in 13 years get  $12*100-10*100=200$ . Hence, find 10-year zero rate from  $r=\ln(200/50)/13=10.6638\%$

Using BBB and CCC: buy 10 of CCC, sell 7 of BBB, Today pay  $10p-700$ , in 13 years get \$300. Hence,  $10p-700=300*\exp(-0.106638*13)$  or  $p=(300*\exp(-0.106638*13)+700)/10=\$77.5$

Alternative solution: Buy  $x$  of AAA,  $y$  of BBB, and sell 1 of CCC such that both dividends and face values cancel out, i.e., have a system of two equations:

$12x+10y-7=0$  and  $x+y-1=0$ . Plug  $y=1-x$  from the latter to the former to find

$12x+10-10x-7=0$ , i.e.,  $x=-3/2$ . Hence,  $y=1-x=5/2$ .

Hence, price of CCC is  $-3/2*115+5/2*100=77.5$

**Question 13:** At the time you bought futures contract for 100 oz of gold, the spot gold price was \$1100/oz and the futures gold price was \$1120/oz. As a result of this transaction, in addition to the money you must deposit on your margin account, you will:

- a) Need to pay \$2000 to the seller of this futures contract
- b) Need to pay \$1000 to the seller of this futures contract
- c) Receive \$2000 from the seller of this futures contract
- d) Receive \$1000 from the seller of this futures contract
- e) Neither pay any money to nor receive any money from the seller of this futures contract

Ans: E

**Question 14:**

Assume Alex wants to hedge his expected sale of gold in 1 month with 2-month futures contract while Bob wants to hedge his expected purchase of gold at the same time. Let  $B_t$  be the basis at time  $t$  (where  $t$  is measured the number of months from today, i.e., today  $t=0$ ), and  $F_t$  be the price of the futures contract at time  $t$ . In this case, the effective price

that Alex receives for his gold is equal to \_\_\_\_\_ while the effective price that Bob pay for the gold is equal to \_\_\_\_\_

- a)  $F_0 - B_1$ ;  $F_0 - B_1$
- b)  $F_0 + B_1$ ;  $F_0 + B_1$
- c)  $F_0 + B_1$ ;  $F_0 - B_1$
- d)  $F_0 - B_1$ ;  $F_0 + B_1$
- e)  $F_1 - B_0$ ;  $F_1 - B_0$
- f)  $F_1 + B_0$ ;  $F_1 + B_0$
- g)  $F_1 + B_0$ ;  $F_1 - B_0$
- h)  $F_1 - B_0$ ;  $F_1 + B_0$

**Ans: B**

**Question 15:** Assume you want to hedge your purchase of 600oz of gold in April but contracts are available only with delivery in March or June. Contract size = 100oz. Which of the following is the best hedging strategy?

- A) Buy 6 March contracts
- B) Buy 4 March contracts and 2 June contracts
- C) Buy 3 March contracts and 3 June contracts
- D) Buy 2 March contracts and 4 June contracts
- E) Buy 6 June contracts

**Ans: E**