MA103 Mock Midterm

Name: ______________________________

Time Allowed: 85 minutes
Total Value: 70 marks
Number of Pages: 9

**** Mock tests are meant as a means of providing an extra set of practice questions and basis for a review class. Do not study for the midterm based solely on the topics covered by the mock test! Go back through notes/assignments/homework to ensure you have reviewed all concepts discussed in the course.

Instructions:

- optional, but recommended – create one page of study notes ahead of time, to use while completing the mock test
- download a copy of the mock test – either print a paper copy of the test, or use a pdf annotator (or MS OneNote) and a tablet/iPad
- complete the problems under "test-like" conditions:
  - restrict yourself to a 1.0 to 1.5 hour period to work on the problems
  - attempt the problems without using any reference material, other than your page of study notes (of course, this "cheat sheet" is not allowed for your actual midterm, but is allowed for mock tests to aid in your study process)
  - use only the specific model of calculator allowed for your actual exam: Casio FX-300MS Plus
- provide complete solutions to the mock test questions; all numerical answers should be given as exact values, and not as calculator approximations – regardless of the format for your actual midterm, we use traditional types of problems for the mocks so that we can provide feedback on your understanding of the concepts/methods involved
- scan the paper copy of your work (preferably using the method that has been outlined for your math labs), or save your electronic copy; either way, your completed mock test file should be one PDF, PNG, or JPEG file
- submit your completed mock test file to the appropriate Dropbox on the MaSt page in MyLS, by
  11 p.m. on Wednesday Nov. 4th (for grading and feedback)
- your completed mock test will be graded, and linked in the Feedback section of your Dropbox submission before the scheduled review session.

1. $[6\ marks]$ Write your answer to each of the following questions in the space provided. No justification is necessary.

   (a) $\lim_{x\to\infty} 5^{-1/x} =$ ______________

   (b) $\lim_{x\to-\infty} \left(\frac{e}{\pi}\right)^x =$ ______________

   (c) The domain of the function $f(x) = \sin^{-1} x$ is ______________

   (d) $\cos^{-1}\left(\sin\left(\frac{5\pi}{3}\right)\right) =$ ______________

   (e) If $f(x) = |x+5|$, then f is not differentiable at $x =$ ______________

   (f) If $f(x) = \sin(x)$, then the 103^{rd} derivative, $f^{(103)}(x) =$ ______________

2. $[4\ marks]$ For each of the following, indicate whether the given statement is true (T) or false (F) by writing T or F in the space provided.

(a) If a and b are any real values, then $\ln(ab) = \ln a + \ln b$. Answer: ____________

(b) If $f''(x_0) = 0$, then f has a point of inflection at x_0. Answer: ____________

(c) The graph of a function can have, at most, two horizontal asymptotes. Answer: ____________

(d) L'Hospital's Rule states that $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$, provided the derivatives exist.

Answer: ____________

3. $[11\ marks]$ Evaluate each of the following limits.

(a) $\lim_{x\to 0^+} \left[(1+2x)^{1/x}\right]$

(b) $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$

(c) $\lim_{x \to -\infty} \frac{4x - 1}{2\sqrt{x^2 - 1}}$

4. $[7\ marks]$ Consider the function $f(x) = \begin{cases} \ln(-x) - 3x & , \ x \le -1 \\ \dfrac{|x^2 - x - 2|}{x+1} & , \ x > -1 \end{cases}$.

(a) Determine if f is continuous at $x = -1$. Verify your answer be evaluating appropriate one-sided limits.

(b) Based on our answer to part (a) alone, can we make a conclusion about the differentiability of f at $x = -1$? Explain your answer.

5. $[4\ marks]$ Determine $k'(2)$ if $k(x) = f(g(x)) + 3^x \cdot h(x)$ where f, g and h are all differentiable functions, and being given that:

$$f(2) = 5 \qquad g(2) = 2 \qquad h(2) = -1$$
$$f'(2) = -3 \qquad g'(2) = 1 \qquad h'(2) = -4$$

6. $[5\,marks]$ Consider $f\left(x\right)=2\left(x+1\right)^{x^2}$, $x>-1$. Determine the equation of the tangent line to $y=f\left(x\right)$ at the point $(1,4)$.

7. $[6\,marks]$ Use a linearization, or differentials, to approximate the value of $\sqrt[3]{7.95}$.

8. $[8\ marks]$ Consider the equation: $\sin(x) - 2x = 1 - \dfrac{1}{\sqrt{2}}$

(a) Use the Intermediate Value Theorem to show that the equation has at least one solution in the open interval $\left(-\dfrac{\pi}{4}, 0\right)$.

(b) Use the results from part(a) and Rolle's Theorem to prove that, in fact, the equation has exactly one real solution.

9. $[5\, marks]$ When air expands adiabatically (without gaining or losing heat), its pressure P and volume V are related by the equation:

$$PV^{1.4} = C, \text{ where } C \text{ is a constant.}$$

Suppose that the pressure is decreasing at a rate of 10 kPa/min. Determine the rate of change in volume (in cm^3/min) at the point when pressure is 80 kPa, given that $C = 235\,000$.

10. $[6\ marks]$ Consider the function $f(x) = x\cos x - \sin x,\ x \in [0, 2\pi]$.

(a) Find the absolute extrema of f on $[0, 2\pi]$. Be sure to fully justify your answer.

(b) State, with justification, the range of f.

11. $[10\ marks]$ Consider the function $y = f(x) = \dfrac{10\ln x}{x^2}$.

(a) Determine if the graph of f has any vertical asymptotes and any horizontal asymptotes, justifying your answers by calculating relevant limits.

Consider the function $y = f(x) = \dfrac{10 \ln x}{x^2}$.

(b) Determine on which intervals the graph of f is increasing, and on which intervals it is decreasing.

You may use the fact that $f'(x) = \dfrac{10(1 - 2\ln x)}{x^3}$.

(c) Determine on which intervals the graph of f is concave up, and on which it is concave down.

You may use the fact that $f''(x) = \dfrac{10(6\ln x - 5)}{x^4}$.

(d) Use the information found in parts (a) to (c) to sketch a graph of $y = f(x)$. Label any key features of the graph.