MATH2004A – Test 2 – 1:35 pm - 2:25 pm, Oct. 24
Name: **Student Number:**
Total: 15 marks (for 4 questions). You may write on **both sides**.

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1. [4 points] Consider the polar curve C: $r = 2 + \cos\theta$, $0 \le \theta \le \pi/2$. Find the area bounded by C and the two rays: $\theta = 0$, $\theta = \pi/2$.

Solution: We have [2 points for formula]

$$A = (1/2)\int_0^{\pi/2} (2+\cos\theta)^2 d\theta.$$

So [2 points for evaluation]

$$\begin{aligned} A &= (1/2)\int_0^{\pi/2} (4 + 4\cos\theta + \cos^2\theta) d\theta \\ &= \pi + 2 + (1/4)\int_0^{\pi/2} (1+\cos 2\theta) d\theta \\ &= 2 + 9\pi/8. \end{aligned}$$

2. [3 points] Find the tangent line of the space curve $r(t) =< 2t, t^3, t^2 - 1 >$ at the point with $t = 1$.

Solution:
We have $r(1) =< 2, 1, 0 >$ [1 point].
Next, $r'(t) =< 2, 3t^2, 2t >$.
Then $r'(1) =< 2, 3, 2 >$ [1 point].
The line has the equation $r(t) =< 2, 1, 0 > +t < 2, 3, 2 >$ [1 point].

3. [3 points] Find the distance between the point $P(1, 3, 2)$ and the line $r(t) =< 2, 2, 3 > +t < 1, -1, 2 >$.

Solution:
Take $Q(2, 2, 3)$ on the given line. It is allowed to take a different point on the line.
Then $\overrightarrow{PQ} =< 1, -1, 1 >$. [1 point]
Let R be on the line so that PR is perpendicular to the line.
Let $v = (1/\sqrt{6}) < 1, -1, 2 >$. (the direction of the line)
Then $Comp_v\, \overrightarrow{PQ} = 4/\sqrt{6}$. [1 point]
Let d be the distance. Then

$$d^2 = |\ \overrightarrow{PQ}\ |^2 - 16/6.$$

So $d = 1/\sqrt{3}$ [1 point].

4. [5 points] Consider the two lines:

$$L_1 : \quad \frac{x-1}{1} = \frac{y}{2} = \frac{z+1}{1}, \qquad L_2 : \quad \frac{x-2}{-1} = \frac{y-3}{1} = \frac{z}{2}.$$

(i)[1 point] Show they are skew lines. (ii)[4 points] Find the distance between them.

Solution.
(i) [1 point] Write L_1:

$$x = 1 + t,\ y = 2t,\ z = -1 + t. \tag{1}$$

Write

$$x = 2 - s,\ y = 3 + s,\ z = 2s. \tag{2}$$

Set

$$1 + t = 2 - s, 2t = 3 + s.$$

This gives

$$t = 4/3,\ s = -1/3,$$

which does not ensure

$$-1 + t = 2s.$$

So they are skew lines.

(ii) We need to find a plane P with normal vector n, such that it contains L_2 and is parallel to L_1.
Denote $v_1 =< 1, 2, 1 >$ and $v_2 =< -1, 1, 2 >$.
Then $n = v_1 \times v_2 =< 3, -3, 3 >$ [2 points; 1 point for each equality].
The equation of the plane P is [1 point]

$$(x - 2) - (y - 3) + z = 0,\ or\ x - y + z + 1 = 0.$$

Take $Q(1, 0, -1)$ on L_1. The distance between Q and P is [if n is wrong but the formula below is correct, give full marks for this step].

$$d = |1 - 0 + (-1) + 1|/\sqrt{1 + 1 + 1} = 1/\sqrt{3}.$$