Faculty of Science »
Department of mathematics and statistics

MAT 1341C Test 1, 2011

3-February, 2011.

Instructor: Barry Jessup

Family Name:______________________

First Name:________________________

Student number:____________________

Multiple choice answers → (1–3)

For the marker's use only → (4–6)

1	
2	
3	
4	
5	
6	
[Bonus] 7	
Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this exam. Read each question carefully.
2. This is a closed book exam, and no notes of any kind are allowed. **The use of calculators**, cell phones, pagers or any text storage or communication device **is not permitted.**
3. Questions 1 to 3 are multiple choice. These questions are worth 1 points each and no part marks will be given. <u>Please record your answers in the space provided above.</u>
4. Questions 4 – 6 require a complete solution, and are worth 6 points each, so spend your time accordingly.
5. Question 7 is a bonus question and is worth 4 points. To earn points here will be more difficult than in questions 1-6.
6. **The correct answer in questions 4–7 requires justification written legibly and logically: you must convince the marker that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
7. Where it is possible to check your work, do so.
8. Good luck! Bonne chance!

1. Which two of the following statements are true?

I. The span of two distinct vectors u and v in $\mathbf{R}^3$ is a plane through the origin.

II. The span of a single vector u in $\mathbf{R}^2$ is a line.

III. A set of vectors $\{u, v, w\} \subseteq X$ spans a vector space X if every $x \in X$ is a linear combination of v and w.

IV. Any spanning set for $\mathbf{R}^2$ contains at least two elements.

V. $\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\right\}$ spans $\mathbf{M}_{2\,2}$.

A. I & III
B. II & IV
C. I & II
D. III & IV
E. III & II
F. I & V

2. Which of the following are subspaces of $\mathbf{M}_{2\,2}$?

$$U = \left\{ \begin{bmatrix} x & x \\ y & x-y \end{bmatrix} \in \mathbf{M}_{2\,2} \mid x,\ y \in \mathbf{R} \right\},$$

$$V = \left\{ \begin{bmatrix} x & x+y \\ y & 2y \end{bmatrix} \in \mathbf{M}_{2\,2} \mid x,\ y \in \mathbf{R} \right\}$$

$$W = \left\{ \begin{bmatrix} x & x \\ y & xy \end{bmatrix} \in \mathbf{M}_{2\,2} \mid x,\ y \in \mathbf{R} \right\}$$

A. U only
B. V only
C. W only
D. U and V only
E. U and W only
F. V and W only

3. Which of the following statements are true?

I. A set $\{u, v, w\}$ of vectors is linearly independent iff for scalars $a, b, c \in \mathbf{R}$, $au + bv + cw = 0$ implies $a = b = c = 0$.

II. A set $\{u, v, w\}$ of vectors is linearly independent iff for scalars $a, b, c \in \mathbf{R}$, $au + bv + cw = 0$ when $a = b = c = 0$.

III. A set $\{u, v, w\}$ of vectors is linearly independent iff u is not a linear combination of v and w.

IV. $\{(1, -1), (1, 1)\}$ spans $\mathbf{R}^2$.

V. $\{(1, 0, 1), (1, 1, 1), (2, 1, 2)\}$ spans $\mathbf{R}^3$.

A. I & IV
B. II & IV
C. I & II
D. III & V
E. III & II
F. I & V

4. Let $v = (0,1,1)$ and $U = \{u \in \mathbf{R}^3 \mid \operatorname{proj}_v u = 0\}$

a) Show that if $u = (x,y,z) \in \mathbf{R}^3$, then $\operatorname{proj}_v u = \dfrac{y+z}{2}(0,1,1)$

b) Find a Cartesian equation for U, i.e., find $a, b, c, d \in \mathbf{R}$ such that

$$U = \{(x,y,z) \in \mathbf{R}^3 \mid ax + by + cz = d\},$$

Give a complete geometric description of U.

c) Is U a subspace of $\mathbf{R}^3$?

d) Find a spanning set for U.

5. Let $\mathbf{F}(\mathbf{R}) = \{f \mid f : \mathbf{R} \to \mathbf{R}\}$ be the vector space of real-valued functions defined on $\mathbf{R}$. Recall that the zero of $\mathbf{F}(\mathbf{R})$ is the function that has the value 0 for all $x \in \mathbf{R}$.

Define three functions in $\mathbf{F}(\mathbf{R})$ by

$$\begin{aligned} f(x) &= 1 + x, \\ g(x) &= x + x^2, \\ h(x) &= x + x^2 + x^3, \end{aligned}$$

and let $W = \operatorname{span}\{f, g, h\}$.

a) Show, using the definition of linear independence, that $\{f, g, h\}$ is linearly independent.

b) If $j(x) = 1 - x^2 + x^3$ show that $j \in W$.

c) Is $W = \operatorname{span}\{f, g, h, j\}$? Explain your answer.

6. State whether the following are true (always), or may be false, in the box after the statement. You must justify your answer: if true, explain why, if not, give an example to show it is false.

a) If V is a vector space with u, v, w in V, and $\{u, v, w\}$ spans V, then $\{u, v\}$ also spans V.

b) If U is a vector space with u, v, w in U, and $\{u, v, w\}$ is linearly independent, then $\{u, v\}$ is also linearly independent.

7. [Bonus] Suppose that u, v, w are non-zero vectors in $\mathbf{R}^4$ such that $u \cdot v = u \cdot w = v \cdot w = 0$. Prove that $\{u, v, w\}$ is linearly independent.

(You may use this page for rough work or solutions that did not fit on previous pages.)