Faculty of Science »
Department of mathematics and statistics

MAT 1341A – Test 2 – 2015

19 October, 2015.

Instructor – Barry Jessup.

Family Name:__________________

First Name:____________________

Student number:_______________

Your multiple choice answers → (1–3)

For the marker's use only → (4–6)

1	
2	
3	
4	
5	
6	
[Bonus] 7	
Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. Read each question carefully, and **answer all questions in the space provided after each question.** For questions 4 to 7, you may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
2. Questions 1 to 3 are multiple choice. These questions are worth 1 point each and no part marks will be given. Please record your answers in the space provided above.
3. Questions 4 – 6 and are worth 6 points each, and part marks can be earned. **The correct answers here require justification written legibly and logically: you must convince the marker that you know why your solution is correct.**
4. Question 7 is a challenging bonus question and is worth 3 points. It is *much* more difficult to obtain marks in the bonus question, so spend your time accordingly. You can earn 100% without attempting Q.7.
5. Where it is possible to check your work, do so.
6. Good luck! Bonne chance!

1. Which of the following are subspaces of $\mathbf{R}^3$?

$$U = \{(x,\ y,\ z) \in \mathbf{R}^3 \mid x - 2y + z = 0\}$$
$$V = \{(x,\ xy,\ y) \in \mathbf{R}^3 \mid x, y \in \mathbf{R}\}$$
$$W = \{(x,\ y,\ z) \in \mathbf{R}^3 \mid 2x - 5z = 0\}$$
$$X = \{(x + y, y, x - 2y) \mid x, y \in \mathbf{R}\}$$

A. Only U and V
B. Only U and W
C. Only W and X
D. Only U, V and W
E. Only U, V and X
F. Only U, W and X

2. It is known that a subspace Y of $\mathbf{R}^{110}$ can be spanned by 96 vectors, and that Y has a linearly independent set with 71 vectors. Then it is always true that:

A. $\dim Y < 71$

B. $\dim Y > 71$

C. $71 < \dim Y \leq 96$

D. $71 \leq \dim Y < 96$

E. $71 \leq \dim Y \leq 96$

F. None of the above is true.

3. Suppose $\{u, v\}$ is a linearly **independent** set in vector space V, and that $w \in V$ is chosen so that $\{u, v, w\}$ is linearly **independent**. Which of the following statements is **ALWAYS** true?

A. $\{u, w\}$ is linearly dependent.

B. $\{v, w\}$ is linearly dependent.

C. $\{u, v\}$ is linearly dependent.

D. $u \in \operatorname{span}\{v, w\}$.

E. $v \in \operatorname{span}\{u, w\}$.

F. $w \notin \operatorname{span}\{u, v\}$.

4. Recall the vector space $\mathcal{P}_2 = \{a + bx + cx^2 \mid a, b, c \in \mathbf{R}\}$ of polynomial functions of degree at most 2, and define

$$W = \{p \in \mathcal{P}_2 \mid p(3) = 0\}.$$

a) Show that $W = \operatorname{span}\{x-3, x^2-3x\}$. *(Hint: recall the Factor Theorem: if p is any polynomial and $p(a) = 0$ for some $a \in \mathbf{R}$, then $p(x) = (x-a)q(x)$ for some polynomial q of degree one less than that of p.)*

b) Explain why W is a subspace of $\mathcal{P}_2$ *without using the subspace test.*

c) Find a basis for W. (You may use without proof the fact proved in class that $\{1, x, x^2\}$ is linearly independent.)

d) Find $\dim W$.

(Remember that you must justify your answers.)

5. Let $\mathbf{M}_{2\,2}$ denote the vector space of 2 by 2 matrices with real entries, and define

$$U = \left\{ \begin{bmatrix} a & b \\ a+b & c \end{bmatrix} \in \mathbf{M}_{2\,2} \mid a, b, c \in \mathbf{R} \right\}.$$

a) <u>Either</u> check that U is closed under addition, <u>or</u> express U in another form so you can simply state a theorem that guarantees that U is a subspace.

(For parts (b) and (c) you may assume that U is a subspace of $\mathbf{M}_{2\,2}$.)

b) Find a basis for U, and hence find $\dim U$.

c) Give a basis for U different from the one you gave in (b).

(Remember that you must justify your answers.)

6. State whether each of the following statements is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you <u>must give an explicit example - with numbers, matrices, or functions, as is appropriate!</u>

- If you say the statement is always true, you must give a clear explanation.

a) $X = \{f \in \mathbf{F}(\mathbf{R}) \mid f(x) \geq -1 \quad \text{for all } x \in \mathbf{R}\}$ is a subspace of $\mathbf{F}(\mathbf{R}) = \{f \mid f : \mathbf{R} \to \mathbf{R}\}$.

ANSWER

b) If V is a vector space and $\{v_1, v_2, v_3\} \subset V$ is linearly independent, then $\{v_1, v_2\}$ is also linearly independent.

ANSWER

6 (cont.).

c) $\left\{ \begin{bmatrix} a & a \\ b & c \end{bmatrix} \in \mathbf{M}_{2\,2} \;\middle|\; a, b, c \in \mathbf{R} \right\}$ is a subspace of $\mathbf{M}_{2\,2}$ of dimension 3.

ANSWER $\boxed{}$

d) If v_1, v_2, v_3 and v_4 are non-zero vectors in a vector space V, and $U = \operatorname{span}\{v_1, v_2, v_3, v_4\}$ then $\dim U = 4$.

ANSWER $\boxed{}$

7. [Challenge/Bonus]

Suppose U and W are two 3-dimensional subspaces of $\mathbf{R}^5$.

Explain carefully why there is a non-zero vector in $U \cap W = \{v \in \mathbf{R}^5 \mid v \in U \text{ and } v \in W\}$.
Hint: Assume that $U \cap W = \{0\}$ and find a contradiction.

Note: You cannot choose U or W. Your explanation must work for all 3-dimensional subspaces U and W of $\mathbf{R}^5$.