1. Let $X = \{(a, b, c) \in \mathbf{R}^3 \mid b\,c = 0\}$. Then,

A. X is closed under addition and X is closed under multiplication by scalars

B. X is closed under addition but X is not closed under multiplication by scalars

C. X is not closed under addition but X is closed under multiplication by scalars

D. $(0, 0, 0) \notin X$ but X is closed under addition

E. $(0, 0, 0) \in X$ but X is not closed under multiplication by scalars

F. None of the other statements is true.

2. Which of the following are subspaces of $\mathbf{R}^3$?

(1) $\{(x,\ x + y,\ x + 2y) \in \mathbf{R}^3 \mid x, y \in \mathbf{R}\}$
(2) $\{(x,\ y,\ z) \in \mathbf{R}^3 \mid x - 2 = y - 3 = z\}$
(3) $\{(x,\ y,\ z) \in \mathbf{R}^3 \mid xyz = 0\}$
(4) $\{(x,\ y,\ z) \in \mathbf{R}^3 \mid x - y - z = 0\}$

A. (1) and (2)
B. (1), (3) and (4)
C. (3) and (4)
D. (1) and (3)
E. (1) and (4)
F. (2) and (3)

3. Which of the following are subspaces of $\mathbf{M}_{2\,2}(\mathbf{R})$?

A. $\left\{ \begin{bmatrix} a & 1 \\ b & b \end{bmatrix} \in \mathbf{M}_{2\,2} \mid a, b \in \mathbf{R} \right\}$

B. $\left\{ \begin{bmatrix} a & b \\ 2a & c \end{bmatrix} \in \mathbf{M}_{2\,2} \mid a, b, c \in \mathbf{R} \right\}$

C. $\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbf{M}_{2\,2} \mid ab = 1;\ a, d \in \mathbf{R} \right\}$

D. $\left\{ \begin{bmatrix} a & b \\ c & 0 \end{bmatrix} \in \mathbf{M}_{2\,2} \mid a, b, c \text{ integers.} \right\}$

E. $\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbf{M}_{2\,2} \mid cd = 0;\ a, b \in \mathbf{R} \right\}$

F. None of the above.

4. Let $W = \{(x, y, z) \in \mathbf{R}^3 \mid x + 5y - 4z = 0\}$.

a) Explain very briefly why W is a subspace of $\mathbf{R}^3$. (*You will not need to use the Subspace Test - use work we did in class.*)

c) Find a spanning set for W.

d) Give a complete geometric description of W.

(Remember that you must justify your answers.)

5. Let $\mathbf{M}_{2\,2}$ denote the vector space of 2 by 2 matrices with real entries, and define

$$U = \left\{ \begin{bmatrix} 0 & a \\ b & 2a \end{bmatrix} \in \mathbf{M}_{2\,2} \mid a, b \in \mathbf{R} \right\}.$$

a) Either check that U is closed under addition, or express U in another form so you can simply state a theorem that guarantees U is a subspace.

(For (b) and (c) you may assume that U is a subspace of $\mathbf{M}_{2\,2}$.)

b) Find a spanning set for U.

c) Give a matrix $A \in \mathbf{M}_{2\,2}$ such that $A \notin U$.

(Remember that you must justify your answers.)

6. State whether each of the following statements is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you must give an explicit example - with numbers!

- If you say the statement is always true, you must give a clear explanation.

a) $X = \{f \in \mathbf{F}(\mathbf{R}) \mid f(2) \leq 0\}$ is a subspace of $\mathbf{F}(\mathbf{R})$

ANSWER $\boxed{}$

b) If v and w are vectors in $\mathbf{R}^2$ and X is a subspace of $\mathbf{R}^3$ with $v + 2w \in X$, then both v and w belong to X.

ANSWER $\boxed{}$

6 (cont.).

c) $\left\{ \begin{bmatrix} a & b \\ a & c \end{bmatrix} \in \mathbf{M}_{2\,2} \mid a, b, c \in \mathbf{R} \right\}$ is a subspace of $\mathbf{M}_{2\,2}$.

ANSWER $\boxed{}$

d) If v and w belong to a vector space V then $\text{span}\{v, w\} = \text{span}\{v + w, v\}$.

ANSWER $\boxed{}$

7. [Bonus] Give the set $U = \{(x-4, x) \mid x \in \mathbf{R}\}$ the *non-standard* operations

$$(x,\ y) \oplus (x',\ y') = (x + x' + 4,\ y + y') \quad \text{(vector addition)}$$

and

$$k \odot (x,\ y) = (kx + 4k - 4,\ ky) \quad \text{(multiplication by scalars).}$$

a) Prove that U is closed under the operation of vector addition defined above.

b) Show that U *has* a zero vector. (i.e. find it and show it works.)