Faculty of Science »
Department of mathematics and statistics

MAT 1341A Test 1, 2013

5-October, 2013.

Instructor: Barry Jessup.

Family Name:____________________

First Name:____________________

Student number:____________________

Enter your multiple choice responses here → (1–3)

For the marker's use only → (4–5)

1	
2	
3	
4	
5	
6 [Bonus]	
Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this exam. Read each question carefully.
2. This is a closed book exam, and no notes of any kind are allowed. The use of calculators, communication devices, or any image or text storage device is not permitted.
3. Questions 1 to 3 are multiple choice. These questions are worth 1 points each and no part marks will be given. <u>Please record your answers in the space provided above.</u>
4. Questions 4 and 5 require a complete solution, and are worth 6 points each, **so spend your time accordingly.**
5. Question 6 is a bonus question and is worth 2 points. To earn points here will be *much* more difficult than in questions 1-5.
6. **The correct answer in questions 4–6 requires justification written legibly and logically: you must convince the marker that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
7. Where it is possible to check your work, do so.
8. Good luck! Bonne chance!

1. Suppose $\{u, v, w\}$ is a set of vectors in a vector space V. Which of the following statements are equivalent to

"$\{u, v, w\}$ is linearly independent."

I. None of the vectors u, v or w is a multiple of any other single vector in $\{u, v, w\}$.

II. If a, b, c are scalars, then $au + bv + cw = 0$ implies $a = b = c = 0$.

III. Both $\{u, v\}$ and $\{v, w\}$ are linearly independent.

IV. If $a = b = c = 0$, then $au + bv + cw = 0$.

V. None of the vectors u, v or w is a linear combination of the other vectors in $\{u, v, w\}$.

A. I & II
B. I & III
C. II & III
D. II & V
E. II & IV
F. III & IV

2. Let $X = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \in \mathbf{M}_{2\,2} \mid a - d = 0 \right\}$. Which of the following is a spanning set for X?

A. $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

F. $\left\{ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$

3. Which of the following are subspaces of $\mathbf{M}_{2\,2}$?

$$U = \left\{ \begin{bmatrix} x & y \\ y & z \end{bmatrix} \in \mathbf{M}_{2\,2} \mid x,\ y,\ z \in \mathbf{R} \right\}$$

$$V = \left\{ \begin{bmatrix} 0 & y \\ -y & 0 \end{bmatrix} \in \mathbf{M}_{2\,2} \mid y \in \mathbf{R} \right\}$$

$$W = \left\{ \begin{bmatrix} x & y \\ z & w \end{bmatrix} \in \mathbf{M}_{2\,2} \mid xw - zy = 0 \right\}$$

$$S = \left\{ \begin{bmatrix} x & y \\ z & -x \end{bmatrix} \in \mathbf{M}_{2\,2} \mid x,\ y,\ z \in \mathbf{R} \right\}$$

A. Only U and V
B. Only U and W
C. Only U, V and W
D. Only V, W and S
E. Only W and S
F. Only U, V and S

4. Let $n_0 = (1, 0, 1) \in \mathbf{R}^3$ and define

$$U = \{v \in \mathbf{R}^3 \mid ||v - n_0|| = ||v + n_0||\}.$$

a) If $v \in \mathbf{R}^3$, show by expanding both sides using the dot product that

$$||v - n_0||^2 = ||v + n_0||^2 \iff v \cdot n_0 = 0.$$

($\star$) Now we know that $U = \{v \in \mathbf{R}^3 \mid v \cdot n_0 = 0\}$, and thus U is a subspace of $\mathbf{R}^3$.

b) Give a complete geometric description of U. (Hint: use ($\star$)!)

c) Find a spanning set for U.

d) Is your spanning set in (d) linearly independent?

5. Let $\mathbf{F}[0,\pi] = \{f \mid f : [0,\pi] \to \mathbf{R}\}$ be the vector space of real-valued functions defined on $[0,\pi]$. Define four functions in $\mathbf{F}([0,\pi])$ by

$$f(x) = 1, \quad g(x) = \cos 2x, \quad h(x) = \cos x, \quad \text{and} \quad k(x) = \sin^2 x, \qquad \forall x \in [0,\pi],$$

and let $W = \operatorname{span}\{f, g\}$.

a) Show that $\{f, g\}$ is linearly independent.

b) Show that $h \notin W$.

c) Use trigonometric identities to show that $k \in W$. (*Hint:* $\cos 2x = \cos(x + x)$.)

6. [Bonus] Let U and V be subspaces of a vector space W such that the subset

$$U \cup V = \{w \in W \mid w \in U \text{ or } w \in V\}$$

is also a subspace of W.

Prove carefully that either $U \cup V = U$ or $U \cup V = V$.

(You may find the following fact useful: If U is not a subset of V and V is not a subset of U, then there exists $u \in U$ with $u \notin V$ and $v \in V$ with $v \notin U$.)