Faculty of Science »
Department of mathematics and statistics

MAT 1341A Test 1 2012

13-October, 2012.

Instructor: Barry Jessup.

Family Name:______________________

First Name:______________________

Student number:______________________

Enter your multiple choice responses here →

For the marker's use only →

1	
2	
3	
4	
5	
6	
[Bonus] 7	
Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this exam. Read each question carefully.
2. This is a closed book exam, and no notes of any kind are allowed. The use of calculators, communication devices, or any image or text storage device is not permitted.
3. Questions 1 to 3 are multiple choice. These questions are worth 1 points each and no part marks will be given. Please record your answers in the space provided above.
4. Questions 4 – 6 require a complete solution, and are worth 6 points each, so spend your time accordingly.
5. Question 7 is a bonus question and is worth 2 points. To earn points here will be *much* more difficult than in questions 1-6.
6. **The correct answer in questions 4–7 requires justification written legibly and logically: you must convince the marker that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
7. Where it is possible to check your work, do so.
8. Good luck! Bonne chance!

1. For what value of c is the set of vectors $\{\ (1, c, 1), (0, 1, -1), (1, 0, 2)\}$ linearly dependent?

A. -2
B. -1
C. -1/2
D. 0
E. 1
F. 2

2. In a vector space V, suppose $\{u, v\}$ is linearly independent and w is such that $\{u, v, w\}$ is linearly *dependent*. Which of the following is **ALWAYS** true?

A. $u \in \operatorname{span}\{v, w\}$
B. $v \in \operatorname{span}\{u, w\}$
C. $w \in \operatorname{span}\{u, v\}$
D. $\{u, u + v, w\}$ is linearly independent
E. $\{u, w\}$ is linearly dependent
F. $\{v, w\}$ is linearly dependent

3. If $\{u, v, w\}$ is a set of vectors in a vector space V, and a, b, and c are scalars, which of the following statements are true?

I. If none of the vectors u, v or w is a multiple of any other vector in $\{u, v, w\}$, then $\{u, v, w\}$ is linearly independent.

II. If $au + bv + cw = 0$ can occur only when $a = b = c = 0$, then $\{u, v, w\}$ is linearly independent.

III. If $a = b = c = 0$ implies $au + bv + cw = 0$, then $\{u, v, w\}$ is linearly independent.

IV. If $au + bv + cw = 0$ implies $a = b = c = 0$, then $\{u, v, w\}$ is linearly independent.

A. III & IV

B. II & IV

C. I & IV

D. II & III

E. I & III

F. I & II

4. Let $\mathbf{F}[0, \pi] = \{f \mid f : [0, \pi] \to \mathbf{R}\}$ be the vector space of real-valued functions defined on $[0, \pi]$. Define three functions in $\mathbf{F}([0, \pi])$ by

$$f(x) = \sin 2x, \quad g(x) = \cos x, \quad \text{and} \quad h(x) = 1, \qquad \forall x \in [0, \pi],$$

and let $U = \operatorname{span}\{f, g, h\}$.

a) Show that $\{f, g, h\}$ is linearly independent.

b) Use (a) to show that $\{f + g, g + h\}$ is linearly independent.

c) Give a spanning set S for U with $S \neq \{f, g, h\}$.

d) What is the dimension of U?

5. Let $w = (-1, 0, 1)$, and $Y = \{v \in \mathbf{R}^3 \mid \operatorname{proj}_w v = 0\}$.

a) Show that $Y = \{(x, y, z) \in \mathbf{R}^3 \mid -x + z = 0\}$;

b) Give a complete geometric description of Y.
Is Y a subspace of $\mathbf{R}^3$?

c) Show that $\{(0, 1, 0), (1, 0, 1)\}$ spans Y.

d) What is the dimension of Y?

6. a) If V is vector space with u, v, w in V, and $\{u, v, w\}$ is linearly independent, show that $\{u, v\}$ must also be linearly independent. (**Do not give an example**: use the definition of independence to show that the statement is true in *every* vector space V.)

b) Give an example of 3 vectors u, v, w in $\mathbf{R}^2$, such that $\{u, v, w\}$ spans $\mathbf{R}^2$, and $\{u, v\}$ also spans $\mathbf{R}^2$.

7. [Bonus]

a) Suppose that u, v are two non-zero vectors in $\mathbf{R}^4$ such that $u \cdot v = 0$. Prove carefully that $\{u, v\}$ is linearly independent.

b) Give an example of two non-zero vectors u and v in $\mathbf{R}^4$ such that $\{u, v\}$ is linearly independent but $u \cdot v \neq 0$.