Faculty of Science »
Department of mathematics and statistics

MAT 1341C –DGD 1– Test 3, 2015

9-March, 2015.

Instructor: Barry Jessup.

Family Name:____________________

First Name:____________________

Student number:____________________

Multiple choice answers →	1	
	2	
	3	
	subtotal	
For the marker's use only →	4 [6pts]	
	5 [4pts]	
	6 [3pts]	
	7 [3pts]	
	[Bonus] 8	
	Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. Read each question carefully, and **answer all questions in the space provided after each question.** For questions 4 to 8, you may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
2. Questions 1 to 3 are multiple choice. These questions are worth 1 point each and no part marks will be given. Please record your answers in the space provided above.
3. The number of points for questions 4 - 7 is indicated above, and part marks can be earned in each. **The correct answers here require justification written legibly and logically: you must convince the marker that you know why your solution is correct.** Question 8 is a bonus question and is worth only 2 points. It is much more difficult to earn points in the bonus question.
4. Where it is possible to check your work, do so.
5. Good luck! Bonne chance!

1. Suppose X is a subspace of $\mathbf{R}^6$, that $X \neq \{0\}$ and that $X \neq \mathbf{R}^6$. Which of the following statements are true?

I. X has a spanning set consisting of 6 vectors.
II. X has a linearly independent subset consisting of 6 vectors.
III. $1 \leq \dim X \leq 5$.
IV. X has a basis that spans $\mathbf{R}^6$.
V. For all vectors u, v, w in X, $au + bv + cw = 0$ implies $a = b = c = 0$.

A. III & II
B. I & IV
C. II & IV
D. III & V
E. I & III
F. I & V

2. Suppose $\{u, v\}$ is a linearly **independent** set in vector space V, and that $w \in V$ is chosen so that $\{u, v, w\}$ is linearly **dependent**. Which of the following statements is **ALWAYS** true?

A. $\{u, w\}$ is linearly dependent.

B. $\{v, w\}$ is linearly dependent.

C. $\{v, u\}$ is linearly dependent.

D. $u \in \operatorname{span}\{v, w\}$.

E. $v \in \operatorname{span}\{u, w\}$.

F. $w \in \operatorname{span}\{u, v\}$.

3. Each statement below is True or False.

- Every system of 2 equations in 2 unknowns has a unique solution.
- The set of solutions of the system consisting of the single equation

$$2x - 3y = 0$$

in the three variables x, y and z is a subspace of $\mathbf{R}^3$.

- There is a linear system in 2 variables which is inconsistent.

Choose the correct sequence from the possibilities below.

A. True, True, False.
B. True, False, True.
C. True, False, False.
D. False, True, True.
E. False, False, True.
F. False, True, False.

4. Let $v_1 = (-1, 1, 1, 1), v_2 = (1, -1, 1, 1), v_3 = (1, 1, -1, 0)$, and let W be the subspace of $\mathbf{R}^4$ defined by

$$W = \operatorname{span}\{v_1, v_2, v_3\}.$$

a) Show that $\{v_1, v_2, v_3\}$ is a linearly independent set.

b) Find a basis of W, and hence find $\dim W$.

c) Is $\{v_1 + v_2, v_1 - v_3, v_2 + v_3\}$ a basis of W?

d) Assuming that $v_4 = (1, 0, 0, 0)$ does not belong to W (you do **not** have to check this), explain why $\{v_1, v_2, v_3, v_4\}$ is a basis of $\mathbf{R}^4$.

5.

a) Find the reduced row-echelon form of the matrix, indicating at each step the row operations you use in the form (for example) "$aR_i + R_j \to R_j$":

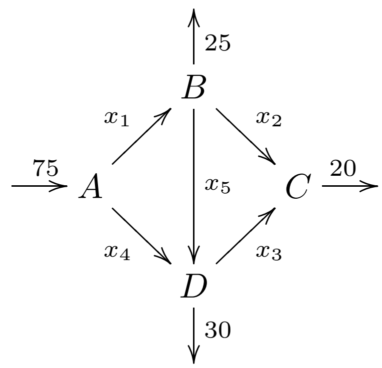
$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 1 & 1 & 0 \\ 2 & -5 & -1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) If the augmented matrix of a linear system in the variables x_1, x_2, x_3 and x_4 is

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & -1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right],$$

find the general solution.

6. Consider the network of streets with intersections A, B, C and D below. The arrows indicate the direction of traffic flow along the one way streets, and the numbers refer to the number of cars observed to enter A or leave B, C and D during one minute. Each x_i denotes the unknown number of cars which passed along the indicated streets during the same period.



Write down a system of linear equations which describes the traffic flow, **together with all the constraints** on the variables x_i, $i = 1, \ldots, 5$. (**Do not solve the linear system.**)

7. State whether each of the following statements is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you <u>must give an explicit example - with numbers!</u>
- If you say the statement is always true, you must give a clear explanation.

a) If V is a vector space and $\{v_1, v_2\} \subset V$ is linearly independent, then $\dim V = 2$.

ANSWER $\boxed{}$

b) If a linear system is consistent, it must be homogeneous.

ANSWER $\boxed{}$

8. [Bonus] Suppose that U and V are subspaces of $\mathbf{R}^{20}$, that $\{u_1, \ldots, u_7\}$ is a basis of U and $\{v_1, \ldots, v_9\}$ is a basis of V.

If there is a non-zero vector $w \in U \cap V = \{w \in \mathbf{R}^{20} \mid w \in U \text{ and } w \in V\}$ prove that $\{u_1, \ldots, u_7, v_1, \ldots, v_9\}$ must be linearly dependent.

(Your proof must work for all subspaces U, V, all choices of bases $\{u_1, \ldots, u_7\}$ of U and $\{v_1, \ldots, v_9\}$ of V, and all vectors w satisfying the condition above: do not choose particular vectors!)

(You may use this page for rough work or solutions that did not fit on previous pages.)