Faculty of Science »
Department of mathematics and statistics

MAT 1341C Version 1 Test 2, 2015

9 February, 2015.

Instructor – Barry Jessup.

Family Name:______________________

First Name:_______________________

Student number:____________________

Multiple choice answers → (1–3)

For the marker's use only → (4–7)

1	
2	
3	
4	
5	
6	
[Bonus] 7	
Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. Read each question carefully, and **answer all questions in the space provided after each question.** For questions 4 to 7, you may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
2. Questions 1 to 3 are multiple choice. These questions are worth 1 point each and no part marks will be given. Please record your answers in the space provided above.
3. Questions 4 – 6 and are worth 6 points each, and part marks can be earned. **The correct answers here require justification written legibly and logically: you must convince the marker that you know why your solution is correct.** Question 7 is a bonus question and is worth 3 points.
4. Where it is possible to check your work, do so.
5. Good luck! Bonne chance!

1. Let $U = \{(x, y, z, w) \in \mathbf{R}^4 \mid x\,y\,w = 0\}$. Then,

A. $(0, 0, 0, 0) \in U$ but U is not closed under multiplication by scalars
B. U is closed under addition and U is closed under multiplication by scalars
C. U is closed under addition but U is not closed under multiplication by scalars
D. U is not closed under addition but U is closed under multiplication by scalars
E. $(0, 0, 0, 0) \notin U$ but U is closed under addition
F. None of the other statements is true.

2. Which of the following are subspaces of $\mathbf{R}^3$?

$U = \{(x,\ y,\ z) \mid 2x - y + 3z = 0\}$
$V = \{(x,\ y,\ z) \mid xy = 0\}$
$W = \{(x,\ y,\ z) \mid 2x = 5z\}$
$X = \{(x,\ y,\ z) \mid x = y + 3 = 7z\}$

A. U and V
B. U, W and X
C. W and X
D. U and W
E. V and X
F. V and W

3. Which two of the following are subspaces of $\mathbf{F}[\mathbf{R}] = \{f \mid f\colon \mathbf{R} \to \mathbf{R}\}$?

$S = \{f \in \mathbf{F}[\mathbf{R}] \mid f(0) = 1\}$
$T = \{f \in \mathbf{F}[\mathbf{R}] \mid f(1) = 0\}$
$U = \{f \in \mathbf{F}[\mathbf{R}] \mid f(0)f(1) = 0\}$
$V = \{f \in \mathbf{F}[\mathbf{R}] \mid f(x) = f(-x), \quad \forall x \in \mathbf{R}\}$

A. S and V.
B. T and U.
C. S and T.
D. T and V.
E. S and U.
F. V and U.

4. Let $W = \{(x, y, z) \in \mathbf{R}^3 \mid x - y + 3z = 0\}$.

a) Explain very briefly why W is a subspace of $\mathbf{R}^3$. (*You will not need to use the Subspace Test - use work we did in class.*)

b) Find a spanning set for W.

c) Give a complete geometric description of W.

(Remember that you must justify your answers.)

5. Let $\mathbf{M}_{2\,2}$ denote the vector space of 2 by 2 matrices with real entries, and define

$$U = \left\{ \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} \in \mathbf{M}_{2\,2} \mid a, b \in \mathbf{R} \right\}.$$

a) Either check that U is closed under addition, or express U in another form so you can simply state a theorem that guarantees that U is a subspace.

(For (b) and (c) you may assume that U is a subspace of $\mathbf{M}_{2\,2}$.)

b) Find a spanning set for U.

c) Give a matrix $A \in \mathbf{M}_{2\,2}$ such that $A \notin U$.

(Remember that you must justify your answers.)

6. State whether each of the following statements is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you must give an explicit example - with numbers, or functic
- If you say the statement is always true, you must give a clear explanation.

a) $X = \{f \in \mathbf{F}(\mathbf{R}) \mid f(x) \leq 0 \quad \text{for all } x \in \mathbf{R}\}$ is a subspace of $\mathbf{F}(\mathbf{R})$

ANSWER $\boxed{}$

b) If u and v are vectors in $\mathbf{R}^2$ and U is a subspace of $\mathbf{R}^2$ with $u - v \in U$, then both u and v belong to U.

ANSWER $\boxed{}$

6 (cont.).

c) $\left\{ \begin{bmatrix} a & a \\ b & c \end{bmatrix} \in \mathbf{M}_{2\,2} \;\middle|\; a, b, c \in \mathbf{R} \right\}$ is a subspace of $\mathbf{M}_{2\,2}$.

ANSWER $\boxed{}$

d) If u, v belong to a vector space V then $\text{span}\{u, v\} = \text{span}\{u, u + v\}$.

ANSWER $\boxed{}$

7. [Bonus] Give the set $U = \{(x-2, x) \mid x \in \mathbf{R}\}$ the *non-standard* operations

$$(x,\ y) \oplus (x',\ y') = (x + x' + 2,\ y + y') \quad \text{(vector addition)}$$

and

$$k \odot (x,\ y) = (kx + 2k - 2,\ ky) \quad \text{(multiplication by scalars).}$$

a) Prove that U is closed under the operation of vector addition defined above.

b) Show that U *has* a zero vector. (i.e. find it and show it works.).