MAT 1341A Test 2 2013

16-November, 2013.

Instructor: Barry Jessup.

Family Name:_____

First Name:_____

Student number:_____

Enter your multiple choice responses here →

1	
2	
3	
subtotal	
4	
5	
6	
7 [Bonus]	
Total	

For the marker's use only →

INSTRUCTIONS

1. You have 80 minutes to complete this exam. Read each question carefully.
2. This is a closed book exam, and no notes of any kind are allowed. The use of calculators, communication devices, or any image or text storage device is not permitted.
3. Questions 4-6 are worth 18 points in total, while questions 1-3 are worth only 3 points in total. *You do not have to answer the questions in the order they are given.*
4. Questions 1 to 3 are multiple choice. No part marks will be given. Please record your answers in the space provided above.
5. **The correct answer in questions 4–7 requires justification written legibly and logically: you must convince the marker that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
6. Question 7 is a bonus question and is worth 3 points. To earn points here will be *much* more difficult than in questions 1-6.
7. Where it is possible to check your work, do so.
8. Good luck! Bonne chance!

1. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and B is a $3 \times n$ matrix then the second row of the matrix AB is

A. the same as the second row of B.

B. the sum of the first and second rows of B.

C. the sum of the second and third rows of B.

D. the sum of the first and third rows of B.

E. the same as the first row of A.

F. the same as the third row of A.

2. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}$. What is the second row vector of A^{-1}?

A. $(-3, 1, 1)$

B. $(5, -3, -11)$

C. $(-1, 1, 0)$

D. $(1, \frac{1}{2}, 1)$

E. $(0, 1, 0)$

F. The matrix A is not invertible.

3 . Find the value(s) of t for which $(2, 6, 5, 2t)$ lies in the subspace of $\mathbf{R}^4$ spanned by $(1, 2, 2, 2)$, $(3, 7, 6, 6)$ and $(1, 2, 1, 2)$.

A. $t = -4$ only.

B. $t = -2$ or -4.

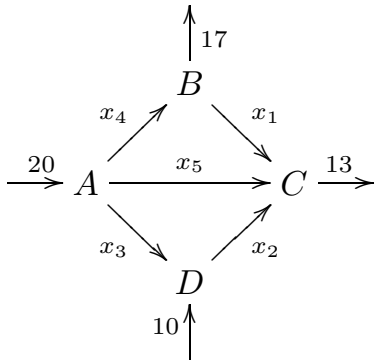
C. $t = 0$ or 2.

D. $t = -2, 0$ or 4.

E. $t = 2$ or 4.

F. $t = 2$ only.

4. Consider the network of streets with intersections A, B, C and D below. The arrows indicate the direction of traffic flow along the one-way streets, and the numbers refer to the exact number of cars observed to enter or leave A B,C and D during one minute. Each x_i denotes the unknown number of cars which passed along the indicated streets during the same period.



a) Write down a system of linear equations which describes the the traffic flow, **together with all the constraints** on the variables x_i, $i = 1, \ldots, 4$. (*Do not simply copy out the equations implicit in (b). You will not get any marks if you do this. Do not perform any operations on your equations: this is done for you in (b)!*)

(Question 4 continued)

b) The reduced row-echelon form of the augmented matrix from part (a) is

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -1 & 0 & -17 \\ 0 & 1 & 0 & 1 & 1 & 30 \\ 0 & 0 & 1 & 1 & 1 & 20 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Give the general solution. (Ignore the constraints at this point.)

c) If $\overline{AC}$ were closed due to roadwork, using your results from (b), find

(i) The maximum flow along $\overline{DC}$, and

(ii) The minimum flow along $\overline{DC}$.

5. Let $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 2 & 2 \end{bmatrix}$.

a) Find the reduced row echelon form of A.

b) Find a basis for $\ker A = \{x \in \mathbf{R}^4 \mid Ax = 0\}$.

(Question 5 continued)

c) Is A invertible? (You do not need to find the inverse, if it exists.)

d) Extend your basis of $\ker A$ to a basis of $\mathbf{R}^4$, if necessary. (Be sure you justify your answer.)

6. Let A and B denote matrices, not necessarily square, and which have more than 1 row and more than 1 column, and let x denote a column vector (i.e., a $k \times 1$ matrix for some k).

State whether each of the following is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you **must give an explicit example - with numbers!** *(Hint: Try an example with 2 or 3 rows or columns.)*
- If you say the statement is true, you must give a clear explanation - by quoting a theorem presented in class, any by giving other valid proof.

a) If A is $m \times n$ and $\operatorname{rank} A = m$, then the system $Ax = 0$ has a unique solution. ☐

b) If $AB = 0$ then either $A = 0$ or $B = 0$. ☐

c) If B has a column of zeros then AB has a column of zeros. ☐

7. [Bonus] Suppose A is an invertible 5×5 matrix and B is any 5×4 matrix with $\operatorname{rank} B = 4$. Prove carefully that $\operatorname{rank} AB = 4$.

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