Faculty of Science »
Department of mathematics and statistics

MAT 1341A Test 2 2012

17-November, 2012.

Instructor: Barry Jessup.

Family Name:____________________

First Name:____________________

Student number:____________________

Enter your multiple choice responses here →

For the marker's use only →

1	
2	
3	
subtotal	
4	
5	
6	
7	
Total	

INSTRUCTIONS

1. Questions 4-7 are worth 21 points in total, while questions 1-3 are worth only 3 points in total. *You do not have to answer the questions in the order they are given.*
2. Questions 1 to 3 are multiple choice. These questions are worth 1 point each and no part marks will be given. <u>Please record your answers in the space provided above.</u>
3. Questions 4—7 require complete solutions. Questions 4–6 and are worth 6 points each and question 7 is worth 3 points, so
   <u>spend your time accordingly.</u>
4. **The correct answer in questions 4–7 requires justification written legibly and logically: you must convince the marker that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
5. **You have 80 minutes to complete this exam. Read each question carefully.**
6. **This is a closed book exam, and no notes of any kind are allowed. The use of calculators, communication devices, or any image or text storage device is not permitted.**
7. Where it is possible to check your work, do so.
8. Good luck! Bonne chance!

1. If C is a $n \times 4$ matrix and $D = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, then the second column of the matrix CD is

A. the same as the second column of C.

B. the sum of the first and second columns of C.

C. the sum of the second and fourth columns of C.

D. the same as the first column of D.

E. the same as the third row of D.

F. the sum of the first and the third columns of C.

2. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 1 & 2 & 2 \end{bmatrix}$. Then the second row of A^{-1} is:

A. $[0 \quad -1 \quad 1]$

B. $[0 \quad -1 \quad 0]$

C. $[2 \quad -1 \quad 0]$

D. $[-1 \quad 1 \quad 1]$

E. $[2 \quad 0 \quad -1]$

F. $[0 \quad 1 \quad 1]$

3. Compute $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{2012}$.

A. $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 0 & -6036 \\ 0 & 1 & 6036 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

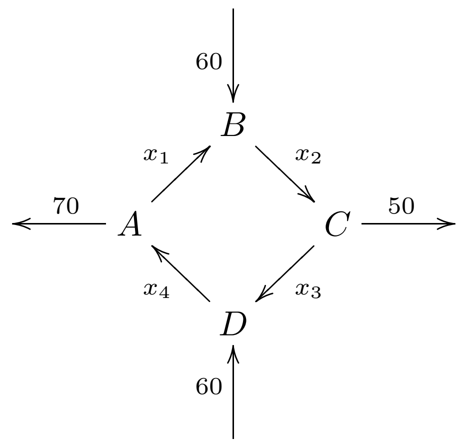
C. $\begin{bmatrix} 1 & 0 & -6036 & 0 \\ 0 & 1 & 0 & 6036 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 & 6036 \\ 0 & 1 & -6036 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 0 & 0 & -3^{2012} \\ 0 & 1 & 3^{2012} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

F. $\begin{bmatrix} 1 & 0 & 0 & 3^{2012} \\ 0 & 1 & -3^{2012} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4. Consider the network of streets with intersections A, B, C and D below. The arrows indicate the direction of traffic flow along the **one way streets**, and the numbers refer to the number of cars observed to enter A or leave B,C and D during one minute. Each x_i denotes the unknown **number** of cars which passed along the indicated streets during the same period.



a) Write down a system of linear equations which describes the the traffic flow, **together with all the constraints** on the variables x_i, $i = 1, \ldots, 4$. (*Do not simply copy out the equations implicit in (b). You will not get any marks if you do this. Do not perform any operations on your equations: this is done for you in (b)!*)

(Question 4 continued)

b) The reduced row-echelon form of the augmented matrix from part (a) is

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -70 \\ 0 & 1 & 0 & -1 & 10 \\ 0 & 0 & 1 & -1 & -40 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Give the general solution. (Ignore the constraints at this point.)

c) Using (b) and the constraints from (a), find the **minimum traffic flows** along the streets

(i) $\overline{BC}$, and

(ii) $\overline{CD}$.

5. Let $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

a) Find the reduced row echelon form of A.

b) Find a basis for $\ker A = \{x \in \mathbf{R}^4 \mid Ax = 0\}$.

(Question 5 continued)

c) Is A invertible?

d) [**Bonus: 2pts**] Extend your basis of $\ker A$ to a basis of $\mathbf{R}^4$, if necessary. (Be sure you justify your answer particularly well.)

6. Suppose A is an $n \times n$ matrix and that,

$$\textit{there is a } \underline{\textit{non-zero}} \textit{ vector } x \in \mathbf{R}^n\textit{, for which } Ax = 0.$$

State whether each of the following is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you <u>must give an explicit example - with numbers!</u> *(Hint: Try a 2×2 example.)*
- If you say the statement is true, you must give a clear explanation - for example by quoting a theorem presented in class.

a) The rank of A is n. $\boxed{}$

b) The columns of A are linearly dependent. $\boxed{}$

(Question 6 continued)

c) There is a vector $b \in \mathbf{R}^n$ such that $Ax = b$ is inconsistent.

7. Let $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, and define a subspace $\mathbf{C} \subseteq \mathbf{M}_{2\,2}$ by

$$\mathbf{C} = \{A \in \mathbf{M}_{2\,2} \mid AJ = JA\}.$$

a) Show that $\mathbf{C} = \left\{ \begin{bmatrix} a & -c \\ c & a \end{bmatrix} \mid a, c \in \mathbf{R} \right\}$.

(Hint: Write $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$*, compute* AJ *and* JA*, and find the general solution to system of the equations in* a, b, c *and* d *that guarantees* $AJ = JA$*.)*

b) Find a basis for $\mathbf{C}$, and hence find $\dim \mathbf{C}$.

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