MAT 1341A Test 2, 2011

19-November, 2011.

Instructor - Barry Jessup

Family Name:_____

First Name:_____

Student number:_____

Enter your multiple choice responses here →

1	
2	
3	
4	
5	
6	
[Bonus] 7	
Total	

For the marker's use only →

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this exam. Read each question carefully.
2. This is a closed book exam, and no notes of any kind are allowed. **The use of calculators**, cell phones, pagers or any text storage or communication device **is not permitted.**
3. Questions 1 to 3 are multiple choice. These questions are worth 1 points each and no part marks will be given. Please record your answers in the space provided above.
4. Questions 4 – 6 require a complete solution, and are worth 6 points each, so spend your time accordingly.
5. Question 7 is a bonus question and is worth 4 points. To earn points here will be *much* more difficult than in questions 1-6.
6. **The correct answer in questions 4–7 requires justification written legibly and logically: you must convince the marker that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
7. Where it is possible to check your work, do so.
8. Good luck! Bonne chance!

1. For what value of α is the set of vectors $\{(1,1,1),(1,2,0),\ (2,3,\alpha)\}$ linearly dependent?

A. -1
B. 2
C. 0
D. 1
E. -1/2
F. -2

2. If A is a 7×12 matrix, what is the smallest possible dimension of the kernel of A?

A. 0
B. 3
C. 5
D. 7
E. 11
F. 12

3. If $B = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 3 & 2 \end{bmatrix}$, then the second row of B^{-1} is:

A. $[1 \;\; 0 \;\; -1]$

B. $[-1 \;\; 0 \;\; 1]$

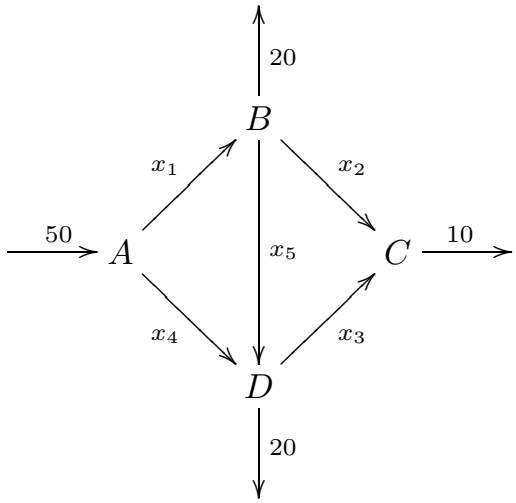
C. $[0 \;\; 1 \;\; -1]$

D. $[2 \;\; 0 \;\; -1]$

E. $[1 \;\; -1 \;\; 0]$

F. None of the above

4. Consider the network of streets with intersections A, B, C and D below. The arrows indicate the direction of traffic flow along the one way streets, and the numbers refer to the number of cars observed to enter A or leave B,C and D during one minute. Each x_i denotes the unknown number of cars which passed along the indicated streets during the same period.



a) Write down the linear system which describes the the traffic flow, **together with all the constraints** on the variables x_i, $i = 1, \ldots, 5$. (Do not perform any operations on your equations: this is done for you in (b). *Do not simply copy out the equations implicit in (b). You will not get any marks if you do this.*)

4b) The reduced row-echelon form of the augmented matrix from part (a) is

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 0 & 50 \\ 0 & 1 & 0 & 1 & 1 & 30 \\ 0 & 0 & 1 & -1 & -1 & -20 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Give the general solution. (Ignore the constraints from (a) at this point.)

c) If $\overline{BD}$ is closed find the maximum and minimum flows along $\overline{BC}$

5. Let $W = \text{span}\{(1,0,1,1), (-1,1,2,0), (1,1,4,2), (0,1,3,1)\}$, and define a matrix A by

$$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 4 & 3 \\ 1 & 0 & 2 & 1 \end{bmatrix}$$

a) Find a basis for W which is a **subset** of the given spanning set above.

b) Extend your basis for W in part (a) to a basis of $\mathbf{R}^4$.

c) Find a basis for $\ker A$, and hence find its dimension.

d) Extend your basis of $\ker A$ in (c) to a basis of $\mathbf{R}^4$.

6. Suppose A is an $n \times n$ matrix and that,

$$\text{there is a vector } b \in \mathbf{R}^n, \text{ for which } Ax = b \text{ is inconsistent.}$$

State whether each of the following is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you must give an explicit example (with numbers!).
- If you say the statment is true, you must give a clear explanation.

a) The system $Ax = 0$ has a unique solution.

b) The matrix A is not invertible.

c) The rows of A are linearly independent.

7 [Bonus]. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and suppose that $ad - bc = 0$. Show carefully that A is not invertible.